PROCEEDINGS

OF THE

American Philosophical Society

HELD AT PHILADELPHIA

FOR

PROMOTING USEFUL KNOWLEDGE

VOLUME XLVII

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VOL. XLVII JANUARY–APRIL, 1908. No. 188.

Stated Meeting January 3, 1908.

Treasurer JAYNE in the Chair.

The decease of the following members was announced:

Dr. Coleman Sellers, at Philadelphia, on December 28, 1907, æt. 80.

Prof. Thomas Day Seymour, at New Haven, on December 31, 1907, æt. 59.

The judges of the annual election of officers and councillors held on this day, between the hours of two and five in the afternoon, reported that the following named persons were elected, according to the laws, regulations and ordinances of the Society, to be the officers for the ensuing year.

President:

William W. Keen.

Vice-Presidents:

George F. Barker, William B. Scott, Simon Newcomb.

Secretaries:

I. Minis Hays, James W. Holland,
Arthur W. Goodspeed, Amos P. Brown.

Curators:

Charles L. Doolittle, William P. Wilson, Leslie W. Miller.

Treasurer:

Henry La Barre Jayne.

Councillors:

(*To serve for three years.*)

Hampton L. Carson, Harry F. Keller,
Talcott Williams, Francis B. Gummere.

Stated Meeting January 17, 1908.

Councillor ROSENGARTEN in the Chair.

A letter was received from the American Institute of Electrical Engineers, inviting the Society to be represented at the Memorial Exercises in honor of Lord Kelvin, to be held on January 12, at 3 P. M.; and from Vice-President Scott announcing the appointment of Mr. Andrew Carnegie and Professor Michael I. Pupin to represent the Society on the occasion.

A letter was received from the Committee of Organization of the First Congress of Chemistry and Physics in memory of the celebrated Russian Chemist, Mendéléeff, announcing that the Congress will be held at the University of St. Petersburg on the second to the twelfth of January, 1908.

The decease was announced of Professor Charles Augustus Young, at Hanover, N. H., on January 3, 1908, æt. 78.

PROFESSOR LEO LOEB read a paper on "Tumor Growth and Tissue Growth." (See page 3.)

TUMOR GROWTH AND TISSUE GROWTH.

By LEO LOEB.

(*Read January 17, 1908.*)

In the course of the last five years, partly through the aid of their respective governments and partly through private initiative, institutions have been founded in the majority of civilized countries for the investigation of the causes and the conditions of growth of malignant tumors; or, as briefly named, for the investigation of cancer. This fact proves more clearly than anything else could do the widespread interest that has recently been aroused in this part of pathological research. Pathological investigations share with those of other sciences a double nature. On the one hand, their problems are of a practical character. Pathology wants to find the causes of diseases and the conditions that favor and inhibit their progress, in order to lay a firm and scientific basis for their cure. In this respect, pathology is an applied, a technical science. On the other hand, pathology desires to analyze the conditions that ultimately lead to death, in order to recognize some of the phenomena of life. In that sense, pathology is a pure science; its aim is philosophical.

Tempting as it might be to relate something of the first attempts of pathology to find the cause and the cure of cancer, I shall here, rather, turn to the purely theoretical aspects of these investigations and indicate some of the results of tumor investigations that have some bearing upon one of the fundamental characteristics of living matter—the ability to grow. Before entering, however, upon a necessarily very limited discussion of some of the relations between tissue and tumor-growth, it might be well to indicate what a tumor is; and, especially, what a cancer is.

Perhaps I can best approach this delicate task by stating some varieties of growth that are not included under the term tumors.

Our bodies consist of cells (that is small parts of protoplasm with nuclear material), of products of cells of different kinds, of decomposition products of cells and of material used for the building up of cells. Here we are concerned with the two former only, namely, with the cells and their direct products. Now growth is based upon an increase in the number or the size of cells in the locality, where growth takes place. The increase in the number of cells can be brought about in two ways: either through the multiplication of pre-existing cells, or through a wandering in of new cells. Cell-growth can take place under various conditions. If toxic substances—the products of bacteria for instance—or even if inert substances foreign to the body are introduced into the organism, a certain proliferation of the neighboring cells and immigration of cells from the blood- and lymph-vessels take place. After a certain period, such reactions come to a standstill, and scar tissue develops. Such a cell-proliferation we do not call a true tumor; but we class it among the inflammatory reactions.

There are other conditions in which an unusual cell-proliferation takes place in the adult organism; in cases of wound healing. If, for instance, a wound is made in the skin, the cells of the epidermis proliferate until the wound is closed; then the additional proliferation ceases. We call this regenerative growth. It lasts only as long as the continuity of the epidermis is interrupted. This is not tumor-growth.

We now come to a third variety of cell-proliferation, distinct from the two former varieties. If a follicle of the ovary ruptures at the time of menstruation, the follicle cells enlarge, and proliferate much more extensively than would be necessary in order to insure wound-healing. There is formed a new growth, which exists for a limited period and then disappears. A still more striking example of this new formation was found in our laboratory in the course of the past year. If, at a certain period after copulation has taken place, or at the period of heat, the inner surface of the uterus is sufficiently exposed and cuts are made in the wall of the uterus, we find that, instead of the ordinary wound-healing, another process takes place, namely: the development of nodules of new tissue, which resembles closely the maternal part of the placenta—without,

however, an ovum being in this case responsible for the new formation; but also in this case the experimentally new-formed decidua, as we call this tissue, dies.

The latter variety of growth resembles much more closely the real tumor-growth than do the former; but in this case also the cell-proliferation, and even the life of the newly formed cells, cease, when the cause for the proliferation has disappeared. The cause for the development of an artificial decidua is probably two-fold: in the first place, a general chemical condition exists in the body at that period; and, under these predisposing conditions, a local stimulus suffices to produce the tumor-like growth. These new formations might be called transitory tumors, because they have a definite life-cycle; they grow for some time, and then they disappear.

In real tumors we find a similar but still more marked cell-proliferation; and they do not have such a definite life-cycle. Real tumors do not retrograde usually, and may even grow, more or less, during the lifetime of the bearer. Furthermore, we do not know the cause of their origin, as we do in the case of the transitory tumor. They grow, and we do not know why: If such tumors grow more rapidly, and especially if they grow deep into the surrounding tissue, digesting it, if parts penetrate into the blood- or lymph-vessels and are carried away to distant parts of the body, and here start a new growth, a so-called metastasis, then we call the tumor malignant, or a cancer.

We distinguish different varieties of cancer, according to the tissue or variety of cells from which these cancers originate. The malignant tumors derived from epithelial surfaces or gland cells, we call carcinomata and the malignant tumors derived from the connective-tissue cells, which unite the functionally more highly developed cells, we call sarcomata. But from whatever tissue these malignant tumors are derived, their main characteristics are identical.

During the second half of the last century, pathologists studied very carefully the microscopical character of the different tumors; and they determined quite accurately the genesis of these tumors from normal tissues. They observed how cells began to grow down into the adjoining tissues in cancer; they described the general spreading out of the new formation, and the character of the sec-

ondary growth; they also determined that a certain number of tumors apparently originate in tissue that has been misplaced during embryonic development. In other cases, long irritation, and occasionally a traumatism, may be held responsible for the origin of cancer. Apparently, however, no further progress could be made by these means of observation. The investigations seemed to have arrived at a dead point.

After a few isolated previous attempts, mainly since the year 1899, the attention of the investigators was directed to the occurrence of tumors in animals; to the fact that cancer in animals frequently occurs endemically. This means that a number of animals are affected with cancer simultaneously in a certain locality. Furthermore, they observed that certain kinds of tumors are characteristic for certain species of animals; and that the tumors occurring endemically in a species of animals are all of the same type.

The most important fact, however, which was fully developed only within the last eight years, is that it is possible to transplant a certain number of cancers into other animals of the same species. Many attempts have been made to transplant cancers into animals of other species and make them grow in these animals, but without any success. A certain kind of cancer found in the dog can be made to grow in some related species, as, for instance, in the fox. Other tumors found in white rats may be transplanted into hybrids between white and gray rats, and the cancer of white mice can occasionally be made to grow in gray mice. The cancer of a Japanese mouse could not be successfully transplanted into white mice, however, but only into the Japanese mice. No such tumors can be transplanted into more distantly related animals, nor can the cancer of man be transplanted into lower animals. A very malignant tumor from a mouse can occasionally be made to grow for a few days in a rat, but the growth soon stops. In a similar way, normal tissues of the body, for instance the epithelium, may be transplanted into other animals of the same species, and kept there alive after an initial growth; but if transplanted into an animal of another species, it grows for a short period and then it dies.

Some tumors, and probably the majority of them, can be transplanted only into the same animal in which they have originated.

Here they live, and even grow; while in other animals of the same species, they die very soon after transplantation. This probably applies to most of human tumors. The same holds good of certain animal tissues and organs; as, for instance, the ovary. They can much more easily be transplanted into the animal of which they have formed an integral part, than into other animals of the same species.

There exists another point of similarity between the transplantation of normal tissues and organs, on the one hand, and of tumors, on the other: in both cases, after transplantation, only the peripheral parts of the transplanted piece usually remain alive; the central part, which is not well supplied with lymph or blood from the host, soon dying. This similarity between the behavior of normal tissues and of tumors after transplantation can be easily explained, if we consider that in both cases we have equally to deal with the inoculation of cells or tissues from an animal organism; and that the transplanted tumor, as can be readily shown by microscopic examination, grows merely from the transferred tumor-cells themselves, and not from the tissues of the receiving host-animal.

On the other hand, however, there exist also some very interesting differences between the growth of normal tissues and of tumor-tissues after transplantation, the former always growing only very slowly for a time, and then ceasing to grow, or merely remaining alive after transplantation; and the latter continuing to grow rapidly, and sometimes continuing to infiltrate the surrounding host-tissue and to make metastases. Their character is not markedly modified through transplantation. Eight years ago I transplanted a sarcoma of a white rat into more than forty generations, without an appreciable decrease in the energy of growth of the tumor cells. The fact that it is possible to propagate tissues of the animal body through years and years in other animals of the same species, without any loss of vitality and power of propagation of the tumor-cells, while they would long since have died if they had remained in the animal to which they originally belonged—suggests, it seems to me, a consideration of great biological significance, namely, the question whether our own body-cells are all equally mortal, or whether their death does depend upon their accidental connection with other cells and with

an organism that dies, and because a certain number of cells, especially of nervous character, cannot survive.

The inevitable fate of all metazoan organisms is death; and this conception deeply influenced all our valuations and directions of thought, as Metchnikoff only recently pointed out in his book on the "Nature of Man." Weismann added one consoling idea: not all of our cells must necessarily die, but only the so-called somatic cells; the germ-cells, ova and the sperm-cells, of each individual may propagate forever, may be immortal. The results of the tumor investigations just mentioned may, perhaps, enlarge the number of cells that may remain alive for so long a period that we cannot see the end at present; ordinary somatic cells may propagate through many generations, long after their brother cells that remained in the original organism have been transformed into simple chemical substances, and who can at present deny the possibility that they may have the potentiality of immortality, as well as the germ cells? Thus the work on tumors leads us into different realms of general biology, and opens up new fields that are not without interest.

The experimental work on tumors has given some other results of an unexpected nature. One of the great achievements of the last century was the development of bacteriological technique by which it is possible for us not only to cultivate bacteria on artificial culture-media, but also to influence markedly their behavior, functions, vitality, and virulence. It has been found to be possible to raise the virulence of certain bacteria by inoculating them into animals through several generations; on the other hand, it is possible to decrease their virulence by subjecting them to certain injurious chemical or physical agencies. Such a bacterial culture with artificially decreased virulence has been used as a vaccine; that means, as a substance that, when inoculated into human beings or animals, without causing the disease, confers immunity against the virulent bacilli.

In experimenting with tumor cells, the surprising result was obtained that, through successive transplantations, by cutting out pieces of tumor, an artificial stimulus is given to the tumor cells, so that they begin to grow more rapidly and more extensively. In other words, their virulence has been increased. This is due to a direct stimulating action upon the tumor cells, and not to secondary

conditions. This explains a fact very familiar to surgeons; namely, that after an operation a recurrent tumor is frequently more malignant than the original tumor.

But it is also possible to decrease the power of propagation of tumor cells without killing them by exposing the cells to chemical and physical injurious influences, in a way similar to that pursued in the case of bacteria. Here, also, we may, not without some hope, look forward to the preparation of some vaccine that may, some day in the future, help us to combat the dreaded disease. Even in this case, however, tumor tissue probably differs only in degree, and not in principle, from normal tissue. At least, this conclusion is indicated by the fact that such an organ as the normal thyroid gland may, without being entirely destroyed, be markedly weakened in its power of growth through a short exposure to the air before transplantation.

There exist, however, some interesting differences of another kind between tumor tissue and normal tissues or organs. Normal organs have a specific metabolism and, in connection with or as a part of this metabolism, they exert distinct specific functions. We understand by functions those physical and chemical processes which attract our attention by their real or apparent significance for the organism as a whole. The normal female mammary gland, for instance, secretes milk under the influence of certain chemical stimuli which are present in the circulation at the end of pregnancy; and it also grows during pregnancy, under the influence of similar stimuli. If we now transplant the mammary gland of a nonpregnant animal into a pregnant animal, the foreign transplanted gland may secrete milk at the end of pregnancy in a similar way to that of the animal's own gland. The circulating chemical substance exerts the same stimulus upon the transplanted as upon the autochthonous gland, and the transplanted gland responds to the stimulus in the normal way.

There exist certain conditions in which a tumor-like hypertrophy of the mammary gland is found in the white rat. The structure of the gland is slightly modified, but the tumor is not infiltrating. We do not call it a cancer, but a benign tumor—an adenoma. If we transplant such a tumor to another place in the original animal, it

heals, and if the animal becomes pregnant, it begins to grow in the same way as the normal gland, but is no longer able to produce milk. It responds, therefore, only to certain stimuli, but not to others.

If we persist still further and transplant a malignant tumor, a cancer, of the mammary gland, we find that it no longer responds to the stimuli of pregnancy. Such tumors do not seem to assume a more rapid growth, nor do they ever secrete milk. The metabolism of tumors differs, however, only in a greater or less degree from that of the corresponding normal tissues; and the tumor tissue can even still continue to secrete certain substances in a similar way to the normal tissues. This has been observed, for instance, in the case of the tumors of the liver and of the thyroid gland, which latter provides a so-called internal secretion, without which widespread changes would take place in our body. It seems, therefore, in the case of the tumor tissues that there exists a parallelism between its loss of function and its capability to respond to chemical stimuli in the body that normally excite and regulate function and growth.

These observations bring us also nearer to an understanding of tissue growth and tumor growth in general. Just now we mentioned substances of various kinds circulating in the body that regulate the growth of normal tissues and of tumors; but there probably exist a number of such substances. How else could we explain the fact that the majority of tumors may be successfully transplanted into the organism in which the tumor had originated, but not into other individuals of the same species? Evidently there must exist some difference between the chemical composition of the blood and lymph of each individual of one species; and each tissue of one individual is more or less adapted to its own body fluid. Furthermore, we have seen that tissues do not grow in animals belonging to different species; there must, therefore, exist substances regulating growth, which are the same in the same species, but differ in different species. Sometimes, however, certain families of white mice differ among one another to a higher degree than the white mice differ from gray mice.

Such substances, however, can merely regulate the growth of normal tissue and of tumor tissue; they are not able to transform normal tissue into tumor tissue. How the latter transformation is

brought about, we do not yet know; and this is one of the problems that remain before us. Of one fact we may be reasonably certain; namely, that the growth-regulating substances to which we referred just now are, in all likelihood, not the primary factors in the production of tumors. We draw this conclusion because the action of such substances has so far not been shown to be hereditary. They influence the growth as long as they are present. If we liberate tissues or tumors from their influence these substances lose their effect at once or relatively soon. If, however, we are able to transplant certain tumors through forty generations of animals and if the tumors preserve their character as tumors, notwithstanding the individual differences of the different animals into which they are transplanted, then there must be present some factor in or near the tumor cells themselves that constantly stimulates their growth and stirs them restlessly to new activity, until through their activity they destroy their host, and thus prepare their own end. What the character of this local stimulus is, we do not yet know. All the discoveries of organisms that have been announced from time to time were found to be based upon erroneous observations; but that does not exclude the possibility that, after all, a microörganism in intimate relation with the tumor cell is the local stimulus acting on the tumor cell.

There are two discoveries that, in themselves of interest, promise to give us a foothold from which to attack successfully this problem: In the first place the endemic occurrence of tumors among animals, to which we alluded above. Here we can determine whether it is caused by hereditary conditions, or whether it is due to microörganisms or environmental factors. Secondly, the surprising fact we learned three years ago, that if we inoculate one kind of tumor, an epithelial tumor, a carcinoma, into animals, the carcinoma, in a certain number of cases, causes the surrounding connective tissue to assume, likewise, a cancerous growth. We have here, therefore, actually succeeded in producing a new tumor, a sarcoma. Such a fact was entirely unforeseen. It could be discovered only through the experimental method of investigation. The more unexpected a new fact, the more welcome it is; the more it promises to change existing conceptions and to open up new roads, where before no way out could be seen.

Lastly, the first steps have already been taken to find a rational way of curing cancer by procuring immunity in a similar way to that by which we are able to cure a certain number of infectious diseases. Protective sera can not only be prepared against bacteria, against toxins, but also against cells; and probably also against tumor cells. The beginning has been made. Certain tumors in animals have been made to disappear in such a way. Let us hope that the future holds still better results, and that we shall be able to alleviate suffering and to gain a deeper insight into conditions that determine the fate of living matter.

Stated Meeting February 7, 1908.

Councillor ROSENGARTEN in the Chair.

A letter was read from the Fourth International Congress of Mathematics, announcing that the Congress will be held at Rome, April 6–11, 1908.

PROFESSOR EDGAR ODELL LOVETT presented a report on the "Leçon sur l'intégration des Equations différentielles aux dérivées partielles professées, a Stockholm (Février-Mars 1906) Sur l'invitation de S. M. le Roi de Suède par M. V. Volterra, Senateur du Royaume d'Italie, Professeur de Physique Mathématique à l'Université de Rome." He also presented a paper on "Integrable Oases of the Problem of those Bodies in which the Force Function is a Function only of the Mutual Distances."

PROFESSOR HORACE C. RICHARDS and PROFESSOR ARTHUR W. GOODSPEED read a paper on "Recent Advances in Color Photography."

Photographs by the Lumière process were exhibited by Dr. Hartzell and Dr. W. P. Wilson.

Stated Meeting February 21, 1908.

Treasurer JAYNE in the Chair.

DR. J. H. HART read a paper on "Artificial Refrigeration."

Stated Meeting March 6, 1908.

Secretary HOLLAND in the Chair.

Letters were read from the Secretary of the Committee of Organization of the Fourth International Congress of Mathematics, to be held at Rome, Italy, April 6–11, 1908, inviting the Society to be represented at the congress, and Vice-President Simon Newcomb was appointed as the Society's delegate.

Stated Meeting March 20, 1908.

Curator MILLER in the Chair.

The death was announced of Sir Samuel Davenport, of Adelaide, Australia.

DR. GEORGE BYRON GORDON read a paper on "Some of the Results of the University of Pennsylvania Expedition to Alaska, 1907."

Stated Meeting April 3, 1908.

Councillor ROSENGARTEN in the Chair.

Letters were received from the Secretary of the Smithsonian Institution informing the Society that the Institution has learned through the Department of State that the Second International Archæological Congress will be held at Cairo, Egypt, on the date of the Latin Easter, 1909, and requesting that the Institution be apprised of the names of scholars likely to attend the Congress.

DR. LEONARD PEARSON read a paper on "Some Aspects of the Production and Distribution of Milk."

General Meeting, April 23, 24 and 25, 1908.

Vice-President SCOTT in the Chair.

April 23, Afternoon Session.

A letter was received from the College of Physicians, of Philadelphia, inviting the President to be present on April 29, 1908, at

the laying of the corner-stone of the new building of the College. Owing to the absence of the President in Europe, Secretary James W. Holland, M.D., was appointed to represent the Society at the ceremony.

The following papers were read:

"The Law of Oresme, Copernicus and Gresham," by THOMAS WILLING BALCH, of Philadelphia.

"The Dramatic Function of Cassandre in the Oresteia of Æschylus," by PROFESSOR WILLIAM A. LAMBERTON, of Philadelphia.

"Goethe's Private Library as an Index of his Literary Interests," by PROFESSOR WATERMAN T. HEWETT, of Ithaca, N. Y.

"Art and Ethnology," by EDWIN SWIFT BALCH, of Philadelphia.

"Cytomorphosis, A Study of the Law of Cellular Change," by PROFESSOR CHARLES SEDGWICK MINOT, of Cambridge.

"Preliminary Report on the Brains of the Natives of the Andaman and Nicobar Islands," by PROFESSOR E. A. SPITZKA, of Philadelphia (introduced by Professor J. W. Holland).

"Observations regarding the Infliction of the Death Penalty by Electricity," by PROFESSOR E. A. SPITZKA, of Philadelphia (introduced by Professor J. W. Holland).

"The Brain of Rhinochimaera," by PROFESSOR BURT G. WILDER, of Ithaca, N. Y.

April 24, Morning Session.

The following papers were read:

"A Comparison of the Albino Rat with Man in Respect to the Growth of the Brain and of the Spinal Cord," by PROFESSOR HENRY H. DONALDSON, of Philadelphia. (See *Journal of Comparative Neurology and Psychology*, Vol. XVIII, No. 4, 1908.)

"Preliminary Report upon a Crystallographic Study of the Hemoglobins: A Contribution to the Specificity of Vital Substances in Different Vertebrates," by PROFESSORS EDWARD T. REICHERT and AMOS P. BROWN, of Philadelphia.

"Recent Discoveries in the Pathology of Rabies," by MAZYCK P. RAVENEL, M.D., of Madison, Wis.

"The Explosion of the Saratoga Septic Tank," by PROFESSOR WILLIAM PITT MASON, of Troy, N. Y.

"Determination of Dominance in Mendelian Inheritance," by CHARLES B. DAVENPORT, Ph.D., of Cold Spring Harbor, N. Y.

"Inheritance in Protozoa," by PROFESSOR HERBERT SPENCER JENNINGS, of Baltimore.

"The Excretory Organs of the Metazoa: A Critical Review," by PROFESSOR THOMAS H. MONTGOMERY, JR., of Austin, Texas.

"The Classification of the Cetacea," by DR. F. W. TRUE, of Washington.

"Additional Notes on the Santa Cruz Typotheria," by W. J. SINCLAIR, Ph.D., of Princeton, N. J. (introduced by Professor W. B. Scott).

Afternoon Session.

The following papers were read:

"Further Researches on the Physics of the Earth, and especially on the Folding of Mountain Ranges and the uplift of Plateaus and Continents produced by movements of Lava beneath the Crust arising from Secular Leakage of the Ocean Bottom," by DR. T. J. J. SEE, of U. S. Naval Observatory, Mare Island, Cal.

"Stratigraphic Observations in the Vicinity of Susquehanna Gap, North of Harrisburg, Pa., by GILBERT VAN INGEN, of Princeton, N. J. (introduced by Professor W. B. Scott).

"Some Chilean Copper Minerals," by PROFESSOR HARRY F. KELLER, of Philadelphia.

"Progress of Demarcation of the Boundary between Alaska and Canada," by PROFESSOR O. H. TITTMANN, of Washington.

"The Leaf Structures of the Bermuda Sand Strand Plants," by PROFESSOR JOHN W. HARSHBERGER, of Philadelphia.

"The Influence of Heat and Chemicals on the Starch Grain," by PROFESSOR HENRY KRAEMER, of Philadelphia.

"A Contribution to a Knowledge of the Fungi of Pennsylvania; Gasteromycetes," by D. R. SUMSTINE, of Wilkinsburg, Pa. (introduced by Dr. A. E. Ortmann).

April 25, Executive Session.

The pending nominations for membership were read and the Society proceeded to an election, and the teller of election reported

that the following candidates had been elected to membership:

Residents of the United States:

Martin Grove Burmbaugh, Ph.D., Philadelphia.
Walter Bradford Cannon, A.M., M.D., Boston, Mass.
James Christie, Philadelphia.
William Hallock, Ph.D., New York City.
Edward Washburn Hopkins, Ph.D., LL.D., New Haven, Conn.
Leonard Pearson, B.S., V.M.D., M.D., Philadelphia.
Josiah Royce, Ph.D., LL.D., Cambridge, Mass.
Jacob G. Schurman, Ph.D., Ithaca, N. Y.
Charles Henry Smyth, Ph.D., Princeton, N. J.
Herbert Weir Smyth, Ph.D. (Göttingen), Cambridge, Mass.
Henry Wilson Spangler, M.S., Sc.D., Philadelphia.
Edward Anthony Spitzka, M.D., Philadelphia.
John Robert Sitlington Sterrett, Ph.D. (Munich), Ithaca, New York.
Richard Hawley Tucker, Mt. Hamilton, California.
Robert Williams Wood, Ph.D., Baltimore.

Foreign Residents:

Ernest Nys, Brussels.
Albrecht F. K. Penck, Ph.D., Berlin.

Morning Session, 10.30 o'clock.

The following papers were read:

"The Solution of Algebraic Equations in Infinite Series," by PROFESSOR PRESTON A. LAMBERT, of Bethlehem, Pa.

"The Investigation of the Personal Error in Double Star Measures which depend on the Position Angle," by MR. ERIC DOOLITTLE, of Philadelphia.

"Some Results of the Ocean Magnetic Work of the Carnegie Institution of Washington," by DR. L. A. BAUER, Director of the Department of Terrestrial Magnetism, Washington (introduced by President Robert S. Woodward).

"Photographs of Daniel's Comet," by PROFESSOR E. E. BARNARD, of Yerkes Observatory, Williams Bay, Wis.

"Astronomical Photography," by DR. JOHN A. BRASHEAR, of Allegheny, Pa.

"The Completion of the Lunar Theory and the Tables of the Moon's Motion to be made therefrom," by PROFESSOR ERNEST W. BROWN, of New Haven.

"The Relative Advantages of Various Forms of Telescopes for Solar Research," by PROFESSOR GEORGE E. HALE, of Solar Observatory, Pasadena, Cal.

"Problems of Three Bodies on Surfaces," by PROFESSOR EDGAR ODELL LOVETT, of Princeton, N. J.

"A Living Representative of the Most Primitive Ancestors of the Plant Kingdom," by GEORGE T. MOORE, Ph.D., head of the Department of Botany, Marine Biological Laboratory, Wood's Hole, Mass.

Afternoon Session.

The following papers were read:

"The Effect of an Angle in a Wire Conductor on Spark Discharge," by PROFESSOR FRANCIS E. NIPHER, of St. Louis.

"Absorption Spectra of Solutions," by PROFESSOR H. C. JONES, of Baltimore (introduced by Professor Ira Remsen).

"The Effect of Certain Preservatives upon Metabolism," by HARVEY W. WILEY, M.D., of Washington.

"A Vedic Concordance," by PROFESSOR MAURICE BLOOMFIELD, of Baltimore.

"On the Lost Tribes of Israel and the Aryan Ancestry of Jesus and His First Disciples," by PROFESSOR PAUL HAUPT, of Baltimore.

"The Sign and Name for Planet in Babylonia," by PROFESSOR MORRIS JASTROW, JR., of Philadelphia.

"Mediæval German Sculpture in the Germanic Museum of Harvard University," by PROFESSOR KUNO FRANCKE, of Cambridge.

"Notes on Greek Vases in the Museum of Science and Art of the University of Pennsylvania," by PROFESSOR WILLIAM N. BATES, of Philadelphia (introduced by Professor Wm. A. Lamberton).

THE LAW OF ORESME, COPERNICUS AND GRESHAM.

By THOMAS WILLING BALCH.

(*Read April 23, 1908.*)

Among the most certain laws known to economic science is the one that, when two moneys of unequal value are placed in circulation at the same time side by side as currency of the realm, the poorer or cheaper will drive the better or dearer from circulation. This law, though fought over most strenuously in this country within recent years, as if its immutable operation had not been thoroughly demonstrated in past ages of humanity, was known in part at least to the Ancients. Of this there is ample proof in the "Frogs" of Aristophanes. In that play, the foremost comic poet dramatist of Greece places in the mouth of the chorus these lines:

"Oftentimes have we reflected on a similar abuse
In the choice of men for office, and of coins for common use;
For your old and standard pieces, valued and approved and tried
Here among the Grecian nations, and in all the world beside,
Recognized in every realm for trusty stamp and pure assay,
Are rejected and abandoned for the trash of yesterday;
For vile, adulterate issue, drossy, counterfeit and base,
Which the traffic of the city passes current in their place."[1]

In Bohn's Classical Library this passage is thus rendered: "The freedom of the city has often appeared to us to be similarly circumstanced with regard to the good and honorable citizens as to the old coin and the new gold. For neither do we employ these at all, which are not adulterated, but the most excellent, as it appears, of all coins, and alone correctly struck and proved by ringing everywhere, both among the Greeks and the barbarians, but this vile copper coin, struck but yesterday and latterly with the vilest stamps."

In the above quotation it is distinctly shown that the better coins that had been current were driven out and replaced by pieces of inferior value. And as a poetic mind like that of Aristophanes could hardly have understood, much less have discovered such a subtle unwritten law of money, had not some knowledge of it been

[1] Frere's translation.

the common possession of the intellectuals of Greece in the epoch in which he lived, we can infer from Aristophanes's statement of it, that the Grecian states passed through the ups and downs of a change in the standard of value caused by a debasement of the currency.

The same state of affairs existed among the Romans, and the amount of benefits and evils that obtained in the reign of each Roman emperor can in a measure be judged by the greater or less purity of the coinage issued in their respective reigns.

The experiences of the ancient world with money as the mechanism of exchange were largely known to the peoples of the Middle Ages, and they had to discover for themselves at a great and bitter cost that any attempt to debase the currency only results in the good money disappearing from circulation to the ruin of the commonwealth and of its inhabitants, especially of the poorer members.

Three men, exercising three different callings, but all three profound students, and two of them ranking among the scholars of the world, in three different countries, in three distinct periods of time, discovered independently of one another and explained to their respective sovereigns that when into the currency of a country a poorer or cheaper money is injected by the side of a better which is the standard of value, the certain and immutable result will be that the currency of the realm will be debased to the standard of the poorer money. For as it will then be possible to pay debts in either money, people will naturally pay them in the cheaper currency, selling the better money by weight at the premium that it will command in the standard of the poorer currency.

These three men were Nicole Oresme, Bishop of Lisieux in Normandy, who stated this subtle unwritten law of money for Charles the Fifth of France, surnamed the Wise; Nicolaus Copernicus of Thorn in Prussia, the discoverer of the Copernican theory of astronomy, who expounded this same law of the currency for Sigismund the First of Poland; and Sir Thomas Gresham, a noted English merchant, who explained it to Elizabeth of England. It is proper, then, that in honor of these three discoverers of an economic truth that is a precious thing for humanity to know, that this law should be called the Law of Oresme, Copernicus and Gresham.

Oresme and Copernicus each prepared a learned and comprehen-

sive treatise for their respective sovereigns on the practical functions and workings of money, and Gresham wrote a letter to his Queen in which he pointed out to her that good and bad coin could not circulate together. No branch of science arises all developed at one bound from the brain of a single man as Minerva sprang all armed from the head of Jove. It advances by successive degrees, as one scholar after another, armed with the knowledge acquired by his predecessors, develops further what the human race knows of the laws of the universe. And as Hugo Grotius, who assembled from all points of the compass the rules and usages that princes and cities observed in his day in their relations one with another in his monumental work, "De Jure Belli ac Pacis," and gave them a further advance in the trend of a humane and civilized development, has justly been called ever since the father of the science of International Law, so Nicole Oresme and, a greater man than he, Nicolaus Copernicus, for their pioneer work in the exposition of the true rules that govern money as the medium of commercial exchange, have just as truly been described by MacLeod as the Castor and Pollux of monetary science. They both delved into the past experiences in the matter of money of their respective countries, and probably made use of much of what the Greek and the Roman publicists had said on the subject. The work of Grotius first redounded to the advantage of humanity by the application of many of the humane principles that he advocated by their practical adoption by Gustavus Adolphus of Sweden in the terrible Thirty Years War. The light shed by Orèsme and Copernicus on the functions of currency first helped to lighten the burdens of humanity through their application by Charles the Fifth of France and Sigismund the First of Poland. And a generation after the true expounder of our solar and planetary system had prepared his treatise on money, Sir Thomas Gresham likewise, through Elizabeth of England, aided the human race to derive the advantages that are conferred upon society by an honestly maintained measure of value.

The importance of the economic work of Nicole Oresme was first revealed to the world at large in 1862 by William Roscher, professor of political economy in the University of Leipzig. Oresme's master work, "Tractatus De Origine, Natura, Jure et Mutationibus Mone-

tarum," was often referred to before that time. But in every case before Roscher saw Oresme's work in manuscript, the examiners of Oresme's learned and lucid treatise failed to grasp its real importance. When, however, it came under the eye of Roscher, a trained economist, he saw at once the profound significance of the work. Under the title of "A Great French Economist of the Fourteenth Century," Roscher called the attention of the world to Oresme's treatise on money. Two years later the French naturalized Pole, Louis Wolowski, also signalized to his adopted country the work of the fourteenth century economist.[2]

Nicole Oresme, who may be looked upon as the first scholar, so far as we now know, to expound comprehensively money as the mechanism of exchange, was by birth a Norman. He studied at the University of Paris, where he was classed in the Norman nation. At the university, Oresme was reputed to be the most able and learned in his knowledge of the sciences and the fine arts. He translated at the request of Charles the Fifth the "Ethics," "Politics," and other works of Aristotle. He delivered at Avignon on December 24, 1363, before Pope Urban the Fifth and the members of the sacred college a sermon in which he censured the high clergy of France. Charles also commissioned him to translate the Bible, in order that this vernacular version might be opposed to that of the Waldensians.

When Charles the Fifth succeeded to the throne of his ancestors, the French, crushed by what was for those times an enormous debt, were groaning under the weight of the accumulated mismanagement of previous rulers, and the "*royaume des lys*" had shrunk to small proportions before the English invasion, and was fast disappearing in misery and anarchy. Owing to the capture of Charles's father, King John, by the English, Charles was called upon to act as regent. During those years he learnt much which later as king he put to valuable practical use. Reigning from 1364 to 1380 under the title of Charles the Fifth, he was, for his able management of the affairs

[2] "Traictie de la première invention des Monnoies de Nicole Oresme" textes français et latin d'après les manuscrits de la Bibliothèque Impériale et "Traité de la Monnoie de Copernic," texte latin et traduction française publiés et annotés par M. L. Wolowski, membre de l'Institut. Paris, Guillaumin et Cie., 1864.

of his kingdom, justly surnamed the Wise. This honorary title, Charles the Fifth, who was a capable and sagacious man, was entitled for in great measure to the fact that he surrounded himself and relied upon the services of men of first rate ability who had strengthened their natural capacities by hard work, such generals as the Breton, Bertrand du Guesclin, such scholars as the Norman, Nicole Oresme. It was Charles the Wise, too, who, in beginning the first collection of manuscripts in the Louvre, that afterwards became the Bibliothèque Royale, then the Bibliothèque Imperiale, and to-day is known as the Bibliothèque Nationale, was the founder of what is to-day the largest depository of learning in the world.

The chief cause of the unhappy state in which the French people found themselves when Duc Charles became king in 1364 was in large measure due to the tampering by their rulers with the weight of the value of the coins of the realm. Many of the French kings had thought to raise revenue by forcing their people to accept a debased coinage. Of these royal false coiners, Dante flays Philip the Fair (1285–1314) in the *Paradiso* in these words:

> "La si vedra il duol che sopra Senna
> Induce, falseggiando la moneta."[3]

In addition to debasing the coinage, the French sovereigns again and again changed the mint price of gold and silver. In the reign of King John the Second, the value of the *livre tournois* was changed between 1351 and 1360 no less than seventy-one times.[4] And what made the resulting confusion from this unjustified and foolish meddling with the measure of commerce still worse was that sometimes the value of the *livre tournois* was raised and sometimes it was lowered. As a result, far from filling the coffers of the king, this policy prostrated commerce, and the wealth in the realm of France shrank. When Charles the Fifth, upon his father's death, ascended the throne, he called upon Nicole Oresme, in order that he might reform the coinage of France, to shed light upon the confused currency of the kingdom. And thus it was that Oresme prepared his most important work, already referred to, the first comprehensive

[3] "There shall be seen the woe that he shall pour
Along the Seine by debasing the coinage."

[4] Wolowski.

treatise upon money, entitled "Tractatus De Origine, Natura, Jure et Mutationibus Monetarum."

Of this work many manuscript copies of the Latin original were made, and also of a French translation by the author himself under the title "Traictie de la premiere invention des monnoies." This translation was placed as early as 1373 at least in the library collected by the direction of King Charles in the Louvre.

Oresme, in stating the various workings of money as the mechanism of exchange, explained in precious words to his sovereign that, whenever the public currency was altered or tampered with in such a way as to bring into circulation two moneys, bearing the same designation but in reality having two different values, the money of lower value inevitably drove the money of higher value out of circulation. For the merchants found it to their advantage either to melt down the pieces of money that contained the higher amount of metal and to sell the bullion by weight or else to export the high weight coins to other lands. Thus Oresme says: "The rate of exchange and the price of the moneys must be for the kingdom as a law and a firm ordinance which in no way must alter or change." And further in speaking of the ratio of exchange between gold and silver, Oresme points out that the value or proportion in which those metals are exchanged in their natural state, is the rate of exchange that must be maintained between gold and silver currency. For if a given amount of gold is worth twenty times as much silver, then a *livre* of gold would be worth twenty *livres* of silver, a *mark* of gold twenty *marks* of silver. "But always this proportion," he says, "must follow the natural habit or rate of gold to silver, in value." The mutations of the currency are of great peril to the national welfare "for the injury which comes by it," he says, "is not so soon felt nor seen by the people, as it would be by another tax, and nevertheless no such nor similar can be more grievous or greater; and, in addition, gold and silver, by such mutations and changes, shrink and diminish in a kingdom, and in spite of all vigilance and prohibition that may be taken, they go abroad where they are accorded a higher value for, by adventure, men carry more voluntarily their moneys to the places where they know these have a greater value."

The luminous treatise of Oresme on money opened the eyes of King Charles to the disastrous results to a country whose government attempted to alter the basic value of its currency. As regent of France during the captivity by the English of his father, King John the Second, who was captured at Poitiers in 1356, Charles had not escaped the prevailing custom among rulers of that epoch to fill the royal purse by debasing the coins of the realm. In the previous century the great ordinance of 1255, which the States Generals of France, assembled at Paris, obtained from the king, Louis the Ninth, promised sound and stable money for the whole kingdom of France, so that the mark of silver should never produce more than six *livre tournois.* This royal promise was broken again and again by the French sovereigns, and Duc Charles, as regent for his captive father, said the value of the mark should be worth twelve *livre tournois.* This cutting in half of the measure of value was the signal for the great rising at Paris in 1357 under Etienne Marcel, the Prevost of the Paris merchants, and it was with difficulty that the regent reasserted the royal authority in the city.[5] The distracted and poverty-stricken state of the people and the low ebb of the kingly power, reënforced by his practical experiences as regent, caused Charles the Wise, though of a physique so frail that he could not march at the head of his army in those years of strife and peril, yet endowed with a superior mind and seeking the advice of sage advisers, to set himself to reorganize the finances of France. The luminous thoughts expressed in the treatise of Oresme he made his own, and during his reign the weight of the gold currency remained a fixed and unchanged quantity, and that of silver was but very triflingly altered. The resulting stability in the value of money, the measure of commercial exchange, reëstablished the regularity of commercial transactions, and furnished an important element to public prosperity. The resources of the realm augmented and with them the power of King Charles grew. To honor the scholar who had made plain the confusion that resulted from tampering with the standard of value, the money of the realm, King Charles had Oresme elected in 1377 Count Bishop of Lisieux in Normandy. And it was there, two years after the king's death in 1380, that the great

[5] Wolowski.

economist died on July 11, 1382, regretted especially by the scholars of his day.

The economic truths that Oresme so well stated in his treatise on money did not become widely known, for his work was written for his king's information, and Gutenberg had not yet made it possible through printing to give them a wide circulation. The truths that Oresme taught and upon which Charles the Wise acted, to the profit of his kingdom and therefore of himself, became in great measure forgotten. A century and a half after Oresme's death they were rediscovered and restated a second time. In the year 1526, in a Latin treatise entitled "Monete Cudende Ratio," written at the request of Sigismund the First, King of Poland, and his Chancellor, Szydlowiecki, Nicolas Copernicus of Thorn in Prussia, who had elucidated for mankind some of the celestial harmonies, gave to the world an exposition of some of the economic truths. Independently of the work of Oresme, of which the Prusso-Polish scholar knew nothing, Copernicus made clear for his sovereign that two moneys of unequal value could not be kept in circulation at the same time. "Gold or silver," he writes, "marked with an imprint, constitutes the money which serves to determine the price of things that are bought and sold, according to the laws established by the State or the Prince. Money is therefore in some sort a common measure of estimating values; but this measure must always be fixed and must conform to the established rule. Otherwise, there would be, necessarily, disorder in the State: buyers and sellers would at all times be misled, as if the ell, the bushel or the weights did not maintain constant quantity.

. . . . . .

"The establishment of money has necessity for cause. Though in weighing only gold and silver it would have been possible to practice exchanges, those metals, from the unanimous consent of men, being considered everywhere as precious things, nevertheless there would be numerous inconveniences to have to carry always weights along, and, all the world not being apt to recognize at the first glance the purity of gold and silver, it is agreed everywhere to have money marked by government with a stamp designed to show how much each coin contains of gold and silver and to serve as a guaranty to public faith."

Then he explains how the value of metal pieces is changed and depreciated.

"The value of money is depreciated by various causes, either by the change of the name, while the same weight of metal contains a mixture of copper which exceeds the measure desired; or because the weight is wanting, although the mixture has been accomplished in the right proportion; or, what is the worst, because the two vices meet together at the same time. The value of money diminishes of itself by reason of a long service that uses the metal and diminishes its quantity and this reason suffices to cause to be placed in circulation a new money. This necessity is recognized by an infallible sign, when the money weighs notably less than the money intended to be acquired. It is understood that there results a deterioration of the money."

At the time Copernicus prepared his treatise on the money of the realm for his sovereign liege, King Sigismund, the Polish Kingdom included Thorn, Danzig, and a large part of Prussia. But a portion of Prussia, including Konigsberg, had been erected by the treaty of Cracow, concluded in 1525 between Sigismund, King of Poland, and Albert, Margraf of Brandenburg, into a hereditary fief for the benefit of the latter and his male descendants, which the margraf was to hold of King Sigismund. As by this feudal tenure by Margraf Albert of part of Prussia, subject to the overlordship of the Polish king, the two countries were in a sense one, Copernicus, in his treatise on the money of the realm, expounded to his king what measures were necessary in order to restore stability to the much depreciated Prussian money and then maintain the value of the new money on a parity so that it could circulate both in Poland and Prussia. After pointing out how useless it was to attempt to introduce into circulation by the side of a depreciated currency one of greater value, he then explained how the introduction of a cheaper measure of value by the side of a higher one would drive the former from circulation.

"If it does not do to introduce a new and good money, while the old is bad and continues to circulate, a much greater error is committed by introducing alongside of an old currency, a new currency of less value; this latter does not merely depreciate the old, it drives it away, so to speak, by main force."

Then in answer to the argument that a depreciated currency helps the poor, he says:

"We see flourish the countries that possess a good currency, while those that only have a depreciated one, fall into decadence and decline. . . .

"It is incontestable that the countries that make use of good currency shine in all the arts, have better workmen, and have of everything in abundance. On the contrary, in the States which make use of a degraded money, reigns cowardice, laziness and indolence."

In order to remedy the distress to which Prussia had been brought by the falsification and debasement of the currency, and to draw Prussia and Poland closer together by developing their commercial relations, it was necessary to coin two moneys of equal intrinsic value, so that they would circulate concurrently in the two lands. One should bear one one side the royal arms of Poland and on the other those of the Prussian land. The other money should likewise have on one side the royal arms of Poland, but on the other the imprint of the prince, that is, the effigy of the king.

"For the first condition to maintain, is that one and the other currency remain under the royal authority, and that they be current and accepted in the whole kingdom by virtue of the prescription of His Majesty; which would be not of a mediocre importance for the conciliation of public opinion and for reciprocal transactions.

"It would be necessary that these two currencies should be of the same degree of fineness, having a similar real value and a similar nominal value, so that, by vigilant care, the State succeeds to maintan perpetually the regulation which it is now question to establish; it does not belong to princes to obtain any profit from the money that they shall coin; they shall add only so much alloy as may be necessary for the difference between the real value and the nominal value to cover the cost of minting, which will avoid the principal attraction to remelt it.

. . . . . .

"It would be necessary, at the time of the emission of the new money, to demonetise the old and forbid entirely its use, allowing it to be exchanged at the mints, in the just proposition of the intrinsic value. Otherwise it would be labor lost to wish to reestablish good money; the confusion that would ensue would be perhaps even worse than the actual state of affairs. The old money would crush all the advantages of the new."

Then Copernicus explained that gold and silver were the base upon which rested the value of money; and went on to show that in order to maintain them both in circulation the ratio between them must agree with the commercial ratio that existed between them.

"It remains," he went on, "for us to expound the manner of the mutual exchange of gold and silver. In order to pass from the class to the kind and from the simple to the composite, it is necessary first to know the price of pure gold to pure silver. It is known that the same exists between pure

gold and silver, as between gold and silver minted under the same stamp; as also that the same ratio applies to gold coins and gold bars as to silver coins and silver bars, provided that they have the same proportion of alloy and that they represent the same weight."

As Oresme and Copernicus explained to their royal masters that by either debasing or raising the coins of the realm disaster and confusion would follow, so also, at the beginning of Queen Elizabeth's reign, one of her merchants, Sir Thomas Gresham, pointed out to his royal mistress this inflexible unwritten law of money.

Of a Norfolk family, the son of Sir Richard Gresham, who was Lord Mayor of London, Sir Thomas Gresham was born in that city probably in 1519, and died there on November 21, 1579. He was educated at Cambridge University, was a Protestant, and all his life took an active part in commercial affairs, often representing in the Low Countries the commercial interests of England. In 1566 and 1567 he built the Royal Exchange in London. He founded also Gresham College, and provided that the science of astronomy should be taught there.

In a letter to Queen Elizabeth, which is headed "information of Sir Thomas Gresham, Mercer, touching the fall of the exchange, MDLVIII," and which begins, "To the Quenes most excellent maiestye," Gresham says:

"Ytt may pleasse your majesty to understande, thatt the firste occasion off the fall of the exhainge did growe by the Kinges majesty, your latte ffather, in abasinge his quoyne ffrome vi ounces fine to iii ounces fine. Whereuppon the exchainge fell ffrome xxvis. viiid to xiiis. ivd. which was the occasion thatt all your ffine goold was convayd ought of this your realme."

The works on money of these three men, who, independently of one another, expounded to their respective sovereigns the evils resulting to the State from any attempt to debase the coinage, did not become generally known to their contemporaries. However, their discoveries through the influence of their royal rulers gradually made some impress upon mankind, and by the end of the seventeenth century it had become common knowledge among the intellectuals of that day. In a pamphlet published in London in 1696, the Law of Oresme, Copernicus and Gresham, though doubtless the writer did not know directly of their works, is thus stated:

"When two sorts of Coin are current in the same nation of like value by denomination but not intrinsically [that is in commercial value], that which has the least value will be current, and the other as much as possible will be hoarded."

In 1858 the British economist, Henry Dunning MacLeod, called attention to Gresham's statement of this unwritten law of coinage, and suggested that it should be known as Gresham's Law. At the time he did not know of the more elaborate treatises of Oresme and Copernicus on coinage. But when the works on money of those two master economists were revealed through the efforts of Roscher and Wolowski in 1862 and 1864 to the world at large, MacLeod, like a true scholar who wishes to give credit to whom honor is due, suggested that this economic law, a law more powerful than the statutes enacted by the strongest Parliamentary bodies, should be known after all three of its discoverers as the Law of Oresme, Copernicus and Gresham.

During the centuries, many nations in various parts of the world have had abundant experience to learn the futility of attempting to maintain in circulation as currency two moneys at a ratio different from the market or commercial ratio existing at that time between those two kinds of money. In every case where such an effort has been made, the money that is underrated gradually drives that which is overrated from the country. And this nation has had several experiences with this law. Without touching here upon the works of other economic scholars, such as Petty, Locke, Wolowski, Jevons, Leon Say, Horton, Bamberger, Laughlin, White and others, who have added to our knowledge of the unwritten laws that govern money as the medium of exchange, it can be safely said that the more the economic experience of the human race is studied, the more does it become clear that any attempt to tamper with the currency of a nation by injecting a debased money into its measure of value is certain to end in disaster through the working of that natural law of finance, the Law of Oresme, Copernicus and Gresham.

ART AND ETHNOLOGY.

By EDWIN SWIFT BALCH.

(*Read April 23, 1908.*)

Man has been studied in many ways and from many directions: history, language, archeology, anatomy, natural history, geography and other sciences have been called upon in the elucidation of the problems of his history, descent, evolution and origin. The evidence which has been gathered from these many different sources about man and his history may be divided into two classes: that which can be obtained from his own personality or his own remains, a class I do not need to mention again in this paper; and that which can be obtained from what man has produced, and this class of evidence may be subdivided into three sub-classes, namely, written records, implements and art.

The most primarily available evidence in tracing the story of the human race is, of course, written records, and whenever we find written records which we can interpret we speak of history; but when, as in the case of savages, there are no written records, or when, as in the case of Old Mexico, we cannot read the records, the subject changes from history into ethnology and pre-history.

When there are no written records, another class of evidence, that obtained from implements, is largely resorted to by ethnologists. The term "implements," as used in this paper, should perhaps be defined as an abbreviated name for the products of the mechanical arts, without some of which at least no man can live. All modern implements have evolved from primitive beginnings, as, for instance, the twelve-inch shell, which is really the most modern form of the flint arrowhead. Much light has been shed already and more will be shed on the story of man by a comparison of the various implements used in different places and at different times.

The other great class of evidence is art, under which term must be understood the fine arts of sculpture, drawing and painting.

Some use has been made of this class of evidence; nevertheless, it is far below what it should be and usually it is only local in its deductions. There are plenty of treatises relating to the art of the white races, of the modern Europeans, of the Romans, of the Greeks; some on Egyptian art; others on Kaldean art and Assyrian art; some on Old Mexican art and Peruvian art, and so forth. But so little is the subject worked out even locally, that there is practically no special publication about African art or Brazilian art, and it is only within the twentieth century that we find the first serious attempt to trace back the wonderful art of China. As a subject of study, either from an artistic or an ethnological standpoint, the art of the world as a whole is so far almost untouched. Even in such an excellent recent art history as Mr. S. Reinach's "Art Throughout the Ages," one finds that by art he means European art alone and that Hindu art or Chinese art or Mexican art are left out in the cold. Whether art comes from only one center or whether there are several foci of dispersion; what relations, what resemblances, and what differences there are in the art of the world as a whole, is as yet an almost virgin field. If I am not mistaken, only one attempt has been made (by the writer himself) to study and classify art in every district of the globe.

Probably the main reason why art in totality is still so largely unstudied is that it is only recently that art specimens from everywhere have been collected, placed in museums, and made accessible. But, connected with this placing of art specimens in museums, there is a curious fact which shows that the art of the world, at present, appears to hang in a sort of borderland between art and science. The specimens are divided. Some are placed in art museums, others in ethnological museums. For instance, in Philadelphia, art specimens are divided between the Pennsylvania Academy of the Fine Arts and the University Archeological Museum; in Washington, between the Corcoran Gallery and the United States National Museum; in New York, between the Metropolitan Museum and the American Museum of Natural History; in Boston, between the Museum of Fine Arts and the Cambridge Peabody Museum. There is no place where anyone can go and get a comprehensive view of art from all over the world.

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The art of at least half the races of the world has thus found its way into ethnological museums. There it is not yet culled out as art, but the specimens are looked on rather as belonging to the class which can be most briefly called implements. This is not to be wondered at. Ethnologists, as a rule, have not had any special art training. Among artists it is a pretty thoroughly understood thing—and this can be stated only as a dictum and not discussed in this paper—that only a trained artist can criticize art seriously; in fact, the present most prevalent opinions about art are largely the consensus of opinion of the many artist art critics of modern times. Whilst possibly unconscious of this fact, ethnologists are usually aware of their inability to discuss the esthetic qualities of art specimens, and hence, while they frequently study the decorative art of savages, its patterns and its origins they are apt to leave the esthetic qualities of art alone.

Whilst scientists, therefore, generally do not give much thought to the esthetic points of the art specimens in ethnological museums, on the other hand, artists and art critics so far have paid no attention to such arts as African or Australian art. In the overwhelming majority of cases, they are doubtless unaware of the existence of such arts, and if they did know of them they would in many cases despise them, because these arts do not have the qualities of Greek art or Japanese art or French art. Art critics also usually know nothing of ethnology, and certainly care less. It takes a good deal of time and thought and study to learn something of ethnology, and any scientist knows that only a specialist can really give an opinion about it. The result of these somewhat complex conditions is that both ethnologists and art critics have neglected the esthetic arts of perhaps half the races of the world.

It seems as if it should be recognized that the present state of things leaves a gap in knowledge. It is time that this gap should be filled in and that the art of the entire world should be worked out as a whole into its proper divisions and its relations. Practically this will amount to forming a new branch of science, a science which might well be termed comparative art, and it seems just as necessary that there should be a science of comparative art as a science of comparative philology or a science of comparative anat-

omy. It will be a science in which art critics and ethnologists will have to work hand in hand; it will either have to be worked out by trained artists and also by ethnologists, or better still, comparative art must be handled by men who are something of specialists in both fields.

Comparative art should not be confounded with comparative archeology. Although there are points of contact, the fields are different. Comparative archeology is mainly based on the results of digging with the pick and the spade. It includes studies of certain phases of art and architecture, of inscriptions, of implements, and some other things. It does not deal with the work of the Eskimo, or the Australian, or the Ashantee of to-day. It is a study of past things.

Comparative art, on the contrary, must deal, not only with the past, but also with the present. It will not be a study of written inscriptions, nor of implements, but it will be a study of art, and architecture so far as this is a form of the fine arts, and it must be applied to every district of the globe, not only to the remotest past in which there was art, but to the actual present of to-day and to the future. It will deal not only with the art of the Pleistokenes and the Assyrians, of the Chinese and the Aztecs, but also with the art of the tribes now living in the Amazon and Kongo forests, in the islands of the Pacific, and on the shores of the Arctic.

Now I do not wish to claim that the study of art specimens is going to clear up all the problems of ethnology, or do away with other methods of studying man and his history, or anything else of that kind. I only want to say that here is a field still mainly untilled, in which there is much work to be done, and from which, when it is properly plowed up, a valuable crop of scientific data may be expected.

That comparative art will bring up new problems and perhaps alter some theories of the present seems probable. For instance, it was formerly generally accepted that there are five races of men: a white, a yellow, a brown, a red and a black. Then other theories were started: one that there are only three races, a white, a yellow and a black; and another that there are four races, a white, a yellow, a red and a black. A study of the art of the world, however, tends

to make one revert to the older theory of five main races, if indeed it does not point to more than five. For it seems as if there were sufficiently numerous distinct arts, with sufficiently individual racial characteristics, as to necessitate the classifying them provisionally into at least five and possibly more main classes, corresponding to the five or more races of man from which these arts spring.

Let me now give you some concrete examples of how art can help clear up ethnology. Take the Pleistokene men of western central Europe, usually mistakenly called the Cave men. We have no written records from the Pleistokenes, but we have implements and art. Their implements show that they must have lived near the edge of a great ice sheet and that their habits must have been not unlike those of the Eskimo of to-day. But their art tells us a great deal of which the implements give no hint. In the first place Pleistokent arts tells us the fauna amongst which these men lived. It takes us back to a past geological epoch, when the mammoth and the woolly rhinoceros tichorinus roamed over western Europe. It proves and is the only proof that they had domesticated the horse and possibly the dog and that they lived sometimes in habitations not unlike the teepees of the Red Amerinds. In the next place Pleistokene art reveals the fact that these earliest positively known men were unquestionably advanced in some mental characteristics. They had certainly stopped hanging on by their tails. No one who was not distinctly intelligent could possibly have made their sculptures, their drawings and their paintings. Another fact their art shows is that in all probability they were not a Negroid race. Ordinary Bantu art, and also the art of Great Benin, is too unlike Pleistokene art to warrant the belief that its makers could have been blood relations of the Pleistokenes. Certain qualities of Pleistokene art suggest early Greek art, but there are more resemblances which suggest Chinese or Eskimo work, so that the evidence of art, and it is the strongest evidence on the subject, is that the earliest known race was a yellow race.

Take the case of the eastern United States. Mr. Henry C. Mercer, I believe, and many other ethnologists claim that there is no civilization preceding that of the Amerinds or American Indians on this continent. Dr. Charles C. Abbott *per contra* claims that

there is an earlier geological horizon whose argillite implements show there was an earlier race. Unfortunately, there are apparently no art specimens known from the same horizon as these argillite implements. But the lucky finding of a few, only a few, works of art, in undisturbed strata, would tell us positively whether those argillite implements belonged to the Amerinds or whether there really was a previous race. In other words, art would tell us what the implements do not.

Take now the case of the people who inhabit the oceanic fringe of Alaska and British Columbia. I believe ethnologists consider that they are members of the red race of America. But their art raises doubts. Whilst it has certainly some resemblances to the art of Old Mexico and of the United States, it has many more to the art of the brown races of the Pacific. It is more nearly in touch with New Zealand art, with New Guinea art, and so forth, than it is with the art of the rest of America. It shows pretty definitely that, even if these northwestern tribes are not primarily a Pacific island race, yet there must have been some intercourse and some immigration, else they could not produce works of art so similar to those of some of the tribes in the southern Pacific.

Let me give you one more instance. The present art of Japan is an intrusive art which came over from China some fifteen hundred years ago, as is shown by written records. Art critics are only just beginning to find out that it has never risen to the heights reached by its parent art of China. But digging has revealed the fact that there was some more elementary art in Japan which was probably earlier than the Chinese influence. This and some of their own more recent work, their discarded suits of lacquered armor are notable examples, have art qualities which are not Chinese. They are much more in touch with some South Sea art, with that of New Ireland, for instance. The evidence of their art would tend to show that the Japanese were a brown race, who adopted much of Chinese civilization.

To sum up now briefly the gist of this paper, I would submit the following main points:

1. Art is found in every part of the world.

2. Art as a whole has not been studied and examined enough as yet.

3. The art of the whole world should be studied from an esthetic point of view not only locally and individually, but in its broadest relations, in its resemblances and its differences. This branch of science might well be called comparative art.

4. Comparative art, that is the study of the relations in the art of the world, must be done from the esthetic standpoint by persons who are trained art critics.

5. Comparative art, properly worked out, may be expected to throw much light on the story of man.

THE BRAIN OF RHINOCHIMAERA.

By BURT G. WILDER.

(*Read April 23, 1908.*)

Four years ago, to the small but very peculiar and ancient group of shark-like fishes known as Chimeroids, Holocephala and Chismopnea, Garman added a Japanese species, *Rhinochimaera pacifica*. His description of the brain was brief and the figures represented only the general form from the dorsum, venter and side. A well-preserved example recently obtained from Alan Owston of Yokohama enables me to confirm Garman's account as to the general Chimeroid character of the brain, especially the cerebellum and adjoining segments, and as to the extraordinary—probably unique—slenderness of the other regions, due not merely, as in Chimaera, to the elongation of the cerebral crura, but also to the pedunculate condition of the olfactory bulbs, whose tracts or crura equal the cerebral in length. The partial dissection of this brain discloses additional features, as shown upon the colored crayon diagram, viz., (1) The cerebral and olfactory cavities. (2) The complete circumscription of these cavities by walls of moderate thickness at the olfactory bulbs and parts of the cerebral hemispheres, but mostly thin and largely membranous. (3) The olfactory crura have thinner walls than in any brain known to me, and the proper nervous substance seems to be confined to their outer or lateral sides. (4) The roof of the undivided cerebral cavity is wholly membranous; likewise a narrow mesal zone of the floor, but the floors of the hemispheres are connected by a terma ("lamina terminalis") as described by me in *Chimaera* in 1877. (5) Each substantial wall of the cerebral cavity begins as a single broad band which divides into a ventral and a lateral portion as it approaches the hemisphere; this condition has not been observed by me in any other brain. (6) There was found no trace of the *Nervus terminalis* of Locy; nor has it been recognized in any other member of the group.

Even were our knowledge of structure, development and geologic records more complete, and even were there more substantial agreement as to the bearing of the facts upon the affinities, rank and succession of the forms concerned, a detailed description of this brain and a full discussion of the significance of its resemblances and peculiarities would be profitable before a comprehensive society like this only when, as urged by me in this hall three years ago, the concrete foundations of neurology are laid in the primary schools, and when no child reaches the age of ten without exposing for himself, drawing, dissecting and describing the brain of a shark.

CORNELL UNIVERSITY, April 20, 1908.

OBSERVATIONS REGARDING THE INFLICTION OF THE DEATH PENALTY BY ELECTRICITY.

By EDW. ANTHONY SPITZKA, M.D.

(*Read April 23, 1908.*)

A great variety of methods of inflicting the death penalty has been devised by the inventive mind of man. The earlier forms are chiefly characterized by cruelty, by an intense and passionate desire to wreak vengeance and, inflict pain upon the condemned and to instill terror into the minds of onlookers. I will not review the ancient methods in detail. There is the *burning at the stake* by the Romans, Jews, ancient Britons, Chinese and by the Spanish Inquisition; *beating with clubs* in Greece and many African countries; *beheading* by axe and block, the sword and the guillotine; *blowing from a cannon,* either by lashing the condemned to the muzzle or by thrusting him into it as a part of the charge; *boiling* in water, oil, melted sulphur, melted lead; *breaking on the wheel; burial alive; crucifixion,* a lingering method in which death was sometimes hastened by the thrust of a spear or a blow with a club; *crucifrangium,* inflicted on Roman slaves and Christian martyrs by laying the legs of the condemned upon an anvil and fracturing the bones with a heavy hammer; *decimation,* used upon mutinous regiments by shooting every tenth man; *dichotomy* or bisecting the body with a saw; *dismemberment; drawing and quartering; drowning; exposure to wild beasts; flaying alive; flogging; knouting; garroting; impalement;* the *"Iron Maiden"; "peine forte et dure"; poisoning; pounding in a mortar; precipitation* from a great height; the *rack; running the gauntlet; shooting; stabbing; stoning; strangling; suffocating*—in short, men have exercised their utmost ingenuity in devising means for inflicting cruel torture and horrible mutilation upon their victims.

As is well known, more than two hundred offenses were punishable with death in England not so very long ago. In modern times the penalty is now universally limited to murder, treason, piracy and

military offenses. To the credit of William Penn and his companions it may be recorded that in 1675, when they founded Pennsylvania, the statutes prescribing death for all sorts of offenses, grave and trivial, were left behind in darkest England with its Newgate and London Tower, and the only one retained was that of death for aggravated cases of murder.

Electrocution.

In the childhood of the human race lightning and thunder played an important part in the religion and the mental life of the various peoples. Jupiter ruled the world by his thunderbolts. The Norse god Thor with mighty arm wielded the hammer of lightning in combat with the enemies of the gods. Every ancient race and tribe has been awed into humble submission before the powerful divinities imagined to preside among the clouds by this fascinating phenomenon of nature. It is even yet feared by man, for is not its deadliness and its destructiveness demonstrated on every hand?

It is now more than a century and a half ago that Benjamin Franklin, accompanied by his son, went to a field in the neighborhood of Philadelphia as a thunder-storm was approaching and by his famous kite experiment discovered that lightning was, as he shrewdly had surmised, in all respects similar to the frictional electricity which man had produced artificially. In 1760 Franklin erected the first lightning rod upon the house of a merchant named West. Although more than five hundred persons are killed and over eight hundred are injured annually in the United States, Franklin's invention, wherever used, has saved countless lives and vast amounts of property. That the sage Franklin ever foresaw the likelihood of employing this death-dealing and mysterious force in the infliction of capital punishment is apparently not on record.

Electrocution (more properly electrothanasia), compounded from "electro-execution," is the popular name for the infliction of the death penalty by passing through the body of the condemned a current of electricity of sufficient intensity to cause death. The method was first adopted by New York State in 1888 by a law which became effective on January 1, 1889, and which provides how many persons may witness the execution, that a post-mortem examination of the

body of the convict be performed and that the body, unless claimed by relatives, be interred in the prison cemetery with a sufficient quantity of quicklime to consume it.

The first criminal to be executed by electricity was William Kemmler, on August 6, 1890, at Auburn Prison. Since that time over one hundred murderers have been executed in New York State and the method has been adopted by Ohio (1896), Massachusetts (1898), New Jersey (1907), and Virginia (1907–8).

Reports on the earlier cases have been published by Drs. Carlos F. MacDonald, E. C. Spitzka, E. W. Holmes, and with reference to nerve-cell changes, by P. A. Fish.

My own observations are based upon thirty-one electrocutions (in the last six and one half years) at Sing Sing Prison, Auburn Prison, Dannemora Prison and Trenton (State Penitentiary). Of these twenty-five came to autopsy at my hands.

The apparatus consists of a stationary engine, an alternating dynamo capable of generating 2,000 volts, a "death-chair" with adjustable head-rest, binding straps and adjustable electrodes. [At Trenton a 2,400-volt current is taken from the public service wire and lowered to the desired tension by a rheostat.]

The voltmeter, ammeter and switchboard controlling the current is located in the execution-room; the dynamo-room is communicated with by electric signals. Before each execution the apparatus is thoroughly tested. When everything is in readiness the criminal is brought in unfettered and usually unassisted, and seats himself in the chair. His head, chest, arms and legs are secured by broad straps, an electrode thoroughly moistened with saturated salt-solution is affixed to the head, another to the calf of one leg, both electrodes being molded so as to assure good contact. The head is not shaved as is popularly thought.

The application of the current is usually as follows: The contact is made with a high potential (1,800 volts) for 5–7 seconds, reduced to 200–250 volts until a half minute has elapsed; raised to high voltage for 3–5 seconds, again reduced to low voltage until one minute has elapsed, when it is again raised to the high voltage for a few seconds and the contact is broken. The ammeter usually

shows that from seven to ten ampères have passed through the criminal's body.

A second or even a third brief contact is sometimes made, partly as a precautionary measure, but more to completely abolish reflexes in the dead body.

The time consumed by the strapping-in process is usually about forty-five seconds and the first contact is made a few seconds later. In all about 60–70 seconds elapse from the moment the convict leaves his cell until he is shocked to death.

The convicts that I have seen thus dealt with have usually slept soundly the night before, they have entered the room calmly and stolidly, often with a half smile on their lips, some without uttering a word, others repeating a brief prayer, still others with a cheerful good-bye to those present. They usually seated themselves without betraying any signs of fear or trembling, curiously watching the strapping-in process for a while, then sitting erect, looking straight ahead at nothing in particular.

The physician in charge observes the respiratory movements of the prisoner and signals to the electrician at a moment when the lungs contain the minimum quantity of air. At the moment that the contact is made the criminal's body stiffens in a state of tonic muscular spasm, restrained by the straps. This spasm abates somewhat as the voltage is reduced, to again attain its maximum with each raise of voltage. When the current is interrupted the body collapses completely. An examination by the physicians usually fails to elicit any signs of life. Occasionally, there is heard a turbulent, incoördinate, accelerated heart-beat, but apparently limited to the auricular chambers of the heart. In only two cases was there any respiratory effort and this was limited to a single contraction of the thoracic respiratory muscles. An additional brief contact or two regularly abolished these reflex phenomena.

The reason for making the contact at the moment that the convict has expired air from his lungs in the natural course of his breathing is this—and it will explain why certain witnesses of the first electrocution thought that life still existed in Kemmler's body. It must be recalled that there is created a terrifically powerful spasmodic contraction of all muscles, including the sphincters and the glottis.

The closure of the glottis confines whatever air may be in the lungs; upon interrupting the current the body becomes entirely limp, the glottis partly relaxes, the thorax collapses and the contained air rushes through the partly closed glottis. A sound resembling a sigh or half groan may be thus produced upon the body of any dead animal; a little mucus present augments the sound into a gurgle. It is no wonder that inexperienced persons then believe life to be still present.

The death is undoubtedly painless and instantaneous. The vital mechanisms of life, *circulation* and *respiration,* cease with the first contact. Consciousness is blotted out instantly and the prolonged application of the current as it is usually practised by Mr. E. F. Davis, the state electrician of New York, ensures the permanent derangement of the vital functions so that there could be no recovery of these. Occasionally, the drying of the sponges through undue generation of heat causes desquamation or superficial blistering of the skin at the site of the electrodes, but not often. Post-mortem discoloration, or lividity, often appears during the first contact. The pupils of the eyes dilate instantly and remain dilated in death.

The post-mortem examination of "electrocuted" criminals reveals a number of interesting phenomena.

The temperature of the body rises promptly and reaches as high as 120° F. to 129½° F. within twenty minutes in many cases. After the removal of the brain the temperature recorded in the vertebral canal was often over 120° F. The development of this high temperature is to be regarded as resulting from the active metabolism of tissues not (somatically) dead within a body where all vital mechanisms have been abolished, there being no circulation to carry off the generated heat. The maximum heat is generated at the site of the leg-electrodes, where muscle (myosin) coagulation is most extensive. Furthermore, the release of from ten to twenty horse-power of energy within the body must contribute materially to the caloric increase.

The heart, at first flaccid when exposed after death, soon contracts and assumes a tetanized condition. This is particularly marked in the left ventricle; on the whole the organ assumes the form of a heart *in systole.* In one case (Koenig) the right ven-

tricular wall of the heart had ruptured in several places. In one case I was able to elicit slight fibrillar contractions, limited to the small area stimulated, by touching the wall of the heart with a cold instrument. In several cases mechanical irritation of the *atrio-ventricular bundle* elicited slight contractions limited to the columnæ carneæ and the papillary muscles of the left ventricle. In experiments conducted with Professor Coplin upon one of these bodies, this mode of contraction could be called forth by faradaic stimulation, although no response was elicited by direct stimulation. In the same individual it was impossible to elicit any response *via* the nerve system, either through stimulation of the cortex (exposed within about ten minutes), the spinal cord or peripheral nerves, although muscular reflexes could always be called forth by directly stimulating the muscle.

The lungs are usually devoid of blood and weigh only seven or eight ounces avoirdupois each.

The blood is profoundly altered bio-chemically. It is of a very dark, brownish hue, and it rarely coagulates. Either the fibrinogen, or the fibrin-ferment, or both, are destroyed.

The maximum damage is undoubtedly wrought in the nerve system though this is not always manifest. Regarding the histologic changes, reports from various sources vary. There is a general agreement as to the frequent occurrence of capillary hemorrhages, disruptive and destructive for adjacent tissues. In the nerve-cells themselves there appears to be no apparent change, although there must have resulted terrific molecular change. P. A. Fish found vacuoles in one case, but no visible changes in another. Aside from the capillary hemorrhages and the arterial anemia with venous congestion, the brain shows no gross changes of appearance. In a case of accidental death from contact with an alternating current of 1,000 volts for about one half minute, Jellinek found extensive streaks of capillary hemorrhages in the gray substance of brain and spinal cord together with more or less destruction of the nerve cells, extrusion of the cell nucleus, etc.

In the case of Strollo, I have had sections made of the pons, oblongata and spinal cord by my colleague, Dr. Radasch, and these have revealed curious circular areas with a peripheral zone of con-

densation which fades off into the surrounding unaffected areas. The bulk of the central rarified portion shows a delicate network of loose fibrillæ which in all probability are glia fibers. The cellular elements in the rarefied area are few in number though apparently free nuclei are scattered in this portion. These areas follow more or less closely the course of the finer vessels. Many of them contain an unruptured vessel centrally located, while others contain longitudinal sections with the areas arranged in a bead-like manner along such vessel. These areas are larger and more numerous in the pons than in the oblongata and spinal cord and apparently distributed in the longitudinal directions more frequently than in other directions. They seem to resemble gaseous emphysema and are possibly due to the electrolytic liberation of gas in the peri-vascular spaces. One is reminded of the punctures in a piece of paper interposed in the path of the sparks of a static machine.

Through the courtesy of Mr. Wilson H. Brown, Sheriff of Philadelphia, I was permitted to witness a number of hangings and thus was enabled to compare the new method with the old.

The preparations for the execution were always swiftly conducted. Upon this point comparison favors neither method. But after the drop through the trap-door the ensuing seconds and even minutes bear a different tale. In nearly all cases the heart beats for about thirteen minutes. In no case could fracture of a cervical vertebra or rupture of ligaments be determined in the ordinary examination.

In one case only was there no movement of the body after the drop, although the heart beat the usual length of time. This prisoner, a Chinaman, apparently died in syncope or of apoplexy. In others the unconsciousness produced by the first shock of the drop appeared to abate and in several instances there were conscious—or at least semi-conscious—efforts at respiration, efforts to reach the neck where the choking sensation was unbearable, efforts at reaching for a support for the feet manifested by such vigorous efforts that several witnesses fainted at the sight.

They veritably " danced upon the air " until the asphyxia (apnea) became so profound as to blot out consciousness, apparently after one or one and one half minutes in some cases.

The anatomy of hanging has been frequently discussed. A recent publication[1] by Dr. Frederic Wood Jones gives the results of the examination of the bodies of one hundred men executed in Nubia in Roman and Byzantine times. Sixty-two were in one trench, forty in another. They were all adult males, with cords binding the legs and arms trussed to the sides. The hanging rope was still *in situ* on one.

Not a single case of damage to the cervical vertebræ was found. The most commonly found lesion was an oblique opening of the sutures of the skull, so that one portion of the skull, represented by the occipital and temporal bones becomes pulled aside from the other portion, represented by the facial part of the skull and the other temporal bone. The basilar suture in most cases was also disunited. The skulls all gave evidence of blood staining.

This remarkable finding of evidence dating about 2,000 years back, prompted me to examine the head and neck bones of five individuals executed by hanging and sent to the Jefferson Medical College for dissection. In not a single instance could I find a fractured cervical vertebra or a separation of any cranial suture. Death had ensued through strangulation.

The Newgate Calendar and other criminal records are full of instances in which the rope broke and the condemned had to be rehanged and even cases where the head was severed from the body. Furthermore, there are not a few authentic cases of resuscitation and total recovery after hanging.

Compared with hanging as well as other methods, electrocution is the most humane, decent and scientific method of inflicting the death penalty because of its efficiency, quickness and painlessness, and it should be adopted by Pennsylvania as well as every state in the Union. The executions should take place in a building remote from the penitentiaries where other convicts, more or less susceptible to reformation, are confined. The erection of scaffolds in prison corridors or the knowledge on the part of other convicts that an electrocution is in progress has a bad, even brutalizing, effect upon them.

[1] *British Medical Journal*, March 28, 1908.

At the time when objections to the hangman's bungling were most strongly urged in favor of some better method, poisoning by prussic acid as well as chloroform were suggested. With regard to the injection of prussic acid by means of the hypodermic syringe, the Gerry Commission report says:

"This is open to the very serious objection that the use of that instrument is so associated with the practice of medicine, and as a legitimate means of alleviating human suffering, that it is hardly advisable to urge its application for the purposes of legal executions against the almost unanimous protest of the medical profession."

It seems to me that the use of chloroform, first suggested by Wilder in 1870, cheap and efficient as it would be, is open to the same objection. There should be a lively sense of violence, of mysteriously overwhelming power, of potent force and destructive energy attached to the means employed in putting the murderous ruffian out of existence. If any sentimentality is to obtain in relation with capital punishment methods it should not be in favor of the "plug ugly" wielders of the stiletto, black-jack and the ever-ready revolver.

Capital punishment has been abolished in Rhode Island, Maine, Michigan and Wisconsin. Kansas had abolished it but restored it after a negro was burned at the stake for an outrage upon a woman. The states of New York, Colorado and Iowa deemed it wise to reënact the death penalty after it had once been abolished, owing to the increase of crime. (The same experience was met with in Switzerland where the penalty was abolished in 1874 and again established in several cantons in 1879.)

In Louisiana the death penalty is inflicted for assault with intent to kill, arson, burglary and administering poison.

In Delaware and North Carolina arson and burglary are capital crimes.

In Missouri seven crimes are punishable by death; among them are murder, train robbery, arson, perjury in a capital case and mayhem.

In Connecticut the law prescribes the death penalty for placing obstructions on a railroad track.

In Utah the law provides that the condemned may choose between hanging and shooting.

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The question "Is capital punishment justifiable"? has agitated the minds of men ever since the dawn of civilization. Public opinion is never so fickle with regard to any problem of life as this one. My own opinion is a firm conviction in favor of it for those who commit premeditated murder, arson, train-wrecking and bomb-throwing. Society needs this penalty for its own protection and it is authorized to use it. The Mosaic law "Thou shalt not kill" refers to murder and not to legal execution. The fear of death is in most men and it is therefore the most powerful means of intimidation. Optimists may hope to see society organized upon such an enlightened plane that the penalty need not be resorted to—but that time is not yet at hand. In nearly every county or state which abolished the penalty, the subsequent increase in crime aroused a clamor for its reëstablishment.

The opinion is held by some that the penalty fails to act as a deterrent for others. The argument is puerile, for this country at least, inasmuch as only 1.3 per cent. of homicides are convicted. In Germany 95 per cent. are convicted, or, proportionately, thirteen times as many. Were the penalty as rigorously enforced in the case of murder as the whipping-post is used in Delaware for various crimes, its deterrent effects would soon become manifest. It is idle to talk of anything but prompt punishment as a deterrent of crime.

Thus, in New York City, in 1904, there were 147 first degree murders; but there were only 27 convictions and only two were executed. In the same year, in Philadelphia, 48 murder trials resulted in only 7 verdicts of murder in the first degree and several of these, on re-trial, received minor sentences. London, with 6,000,000 inhabitants, had 24 murders; 9 were hanged therefor. Chicago, with 2,000,000 inhabitants, had 128 murders; only 1 was hanged.

The tardy justice meted out to murderers is the most deplorable feature of our legal machinery to-day. There are too many loopholes for escape—long delays, endless appeals, lots of slush about the "unwritten law," numerous legal technicalities and sentimental juries. By the pettifogging of criminal law the great majority of cases are granted new trials in the United States; in Great Britain only 3.5 per cent. Nearly always the appeal is based upon points of pleading and practice and many years elapse before the final settle-

ment of the case. Our administration of justice has degenerated into a sort of "rose-water penology." Its demoralizing effect upon the community is manifested by the rapid increase of crimes of violence among juveniles, so ready to imitate and emulate their seniors in crime. We have become too much accustomed to failure of justice in murder cases. This blot upon our civilization is largely the outcome of our indifference to the way many criminal courts are conducted. Certain magistrates make a farce out of serious business, lawyers wrangle with each other unchecked, witnesses are brow-beaten and bribery and corruption of political complexion degrade the proceedings to the level of a saloon or gambling-den or a policy-shop rather than a court of law.

The explanation is sometimes given that "hard times" influence this appalling increase of crime. That this is not so can be readily shown by reference to statistics. I would rather point to the moral deterioration indicated by the manner in which large sums of money are stolen or used for bribery and corruption and the luxury and reckless extravagance with which some wealthy persons (who ought to be in the penitentiary) offend the decent class of our population. Add to this the manner in which the newspapers set forth the details of brutal crimes and breed familiarity with thoughts of crime.

Society has relaxed too much. The death penalty is a necessity and must not be abolished, else all discipline of society will be relinquished. Though society "revolts at the old religious dogma of the retribution of hell, the church still retains it as essential in its terrible dissuading appeal to the imagination of men" (*New York Sun*). Let us, therefore, in our penology, adhere to what the test of time has proven to be an efficient check if only it be carried out as has been done in Germany and Great Britain.

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PRELIMINARY NOTE ON THE BRAINS OF NATIVES OF THE ANDAMAN AND NICOBAR ISLANDS.

By EDW. ANTHONY SPITZKA, M.D.

(*Read April 23, 1908.*)

Physical anthropology, or comparative human morphology, has been largely based upon cranial configuration. Since the days of Camper and Blumenbach, the classification of the human races is based on more conprehensive morphologic foundations, for with cranial morphology as the first criterion, there have been added criteria derived from the entire skeleton, the soft tissues and the brain. The last-mentioned organ has been the least studied because it is usually most difficult to obtain, preserve and study. Nevertheless, interest in this subject is manifestly increasing among anatomists and anthropologists, for they appreciate the fact that there is a pressing need for fruitful research in anthropologic encephalometry among the exotic races, so rapidly becoming impure or even extinct. Many American Indian tribes have disappeared; the volcanic outbreak in Martinique has wiped out nearly all Caribs. The Australian natives driven to the desiccated wastes of the interior, many African tribes succumbing in the arid deserts, the Eskimos decimated by epidemics of small-pox, measles and pneumonia—all these and many others are dying out and warn us to make haste in recording observations upon them while they still exist.

It has been my good fortune to pursue comparative studies in cerebral morphology based upon the brains of the white race, of Eskimos, Japanese, Chinese, Negroes and Papuans. I am now able to add the brain of a native of the Andaman Islands and one of a native of the Nicobar Islands. For this exceptional privilege I am indebted to the efforts of Dr. W. W. Keen, whose correspondence with Lord Curzon, then Viceroy of India, opened the way to communication with Mr. H. H. Risley, Director of Ethnography for India; Mr. W. R. H. Merk, Superintendent of Port Blair, and Major

A. R. S. Anderson (M. B. Cantab.) I.M.S., senior medical officer at Port Blair, Andamans. To all these I desire to acknowledge herewith my thanks.

The Andaman Archipelago is a group of densely wooded islands about 1,760 square miles in area, situated in the Bay of Bengal about 180 miles southwest of Cape Negrais, Burma, and about 60 miles distant from the more southerly Nicobar Islands. The inhabitants have been considered a most primitive and savage race. Accounts of their cannibalism are found in the ancient Chinese writings and the Andamanese are probably referred to by Ptolemy as the "anthropophagi." Port Blair is a convict settlement and the convicts are deterred from making efforts to escape by their fear of the natives. From the observations of E. H. Man, who, more than any other, has made the race a study, it appears that the Andamanese are Negritos and not Papuans. They are well made and well proportioned. Their skulls are brachycephalic. Their lips are not thick, their profiles are good and they have no peculiar odor like that which is found in the African race. Their extremities are small, but the heel projects slightly to the rear. The average height of the men is 149 cm., of the women 140 cm. The average weight of the men is 98 and 93 pounds respectively. The color of the Andamanese is generally dark bronze or copper color; often the color of soot and even quite black. The hair is woolly, but its cross-section is not always elliptical. In a letter to the Smithsonian Institution, Dr. Abbott says of them: "They are a happy, merry, little people, infantile both in looks and behavior. Unfortunately they are dying out. Contact with civilization is making the women barren and there are comparatively few children."

Mr. Man thinks that it has been pretty well demonstrated that these Negritos in the Andaman Archipelago, so unlike any of their immediate neighbors, are aborigines and have inhabited the group from prehistoric times. The population in 1901, Dr. Anderson writes me, was 2,200, including women and children.

The Andamanese wear no clothing; its place is taken in a measure by necklaces, circlets for the head, garters, bracelets and belts. They live in thatched huts and sleep on mats. Stones are used as anvil and hammer, clam shells as knives. They fashion old barrel

hoops from wrecked ships into jagged knives. The only thing resembling a musical instrument is a wooden shield-like drum upon which the performer keeps time by striking it with his foot. They make some pottery; the base of the pot is in the form of a cup. To this roll after roll is added and the sides built up, the inner and outer surfaces are smoothed off with an arca shell and ornamented with wavy, checkered or striped designs by means of a pointed stick and baked by placing pieces of burning wood both inside and around the vessel. They make cane baskets, wooden trays and buckets. String is made from vegetable fiber (orchid and Anadendron) and used in making harpoon lines, turtle nets, fishing nets, bowstrings coated with wax, lashings, reticules and necklaces. Bows and arrows, harpoons and fish spears are used in hunting. They build outrigger canoes and simple dugouts which are propelled by paddles, or, in shallow water, by poles or the shaft of a turtle harpoon.

Morphologically, the Andamanese form a definite group. The following criteria are given by Duckworth:[1]

Cranial index	82.1
Alveolar index	102.0
Nasal index	50.9
Height index	77.9
Cranial capacity	1,266 c.c.

The skull is small and round, with prominent jaws. It is cryptozygous, muscular ridges are not very prominent, the mastoid processes are small, and the external auditory meatus is shallow. Brow-ridges are not developed, and sexual differences are often obscure in these skulls. The face is prognathous, the prognathism affecting chiefly the alveolar maxillary margin and being therefore subnasal. The chief distinction is the association of a highly brachycephalic skull of small capacity with dwarf stature and prognathism. The lumbo-vertebral index denotes simian affinities. The sacral curve is very slight, indicating a low position among hominidæ. The scapula is the most pithecoid amongst hominidæ with the possible exception of the Bambuté dwarfs of Africa. The proportions of the limb-bones are simian as regards the radio-humeral and the tibio-femoral, but not as regards the intermembral of humero-femoral indices.

[1] Duckworth: "Morphology and Anthropology," Cambridge Biological Series, 1904.

The Brain.

[Preliminary Report.]

The brain is that of an Andamanese named Juran of the tribe called Aka-yere or Aka-Jaro-da. Juran was a male, aged about 45 years, who died at Port Blair of pulmonary tuberculosis on June 30, 1905. The brain was removed by Dr. Anderson about one hour after death and immersed in a mixture of formalin and water. Its

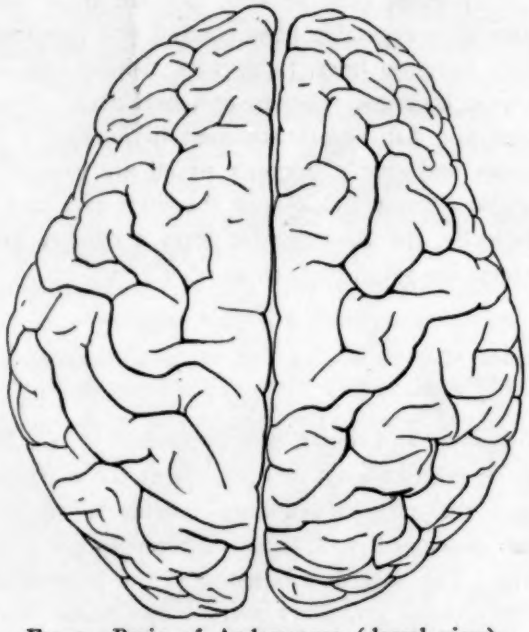

Fig. 1. Brain of Andamanese (dorsal view).

weight while fresh was not noted before being sealed and transported. When received about eight months later it weighed 1,193 grams, as follows:

	Grams.
Left hemicerebrum	532
Right hemicerebrum	525
Cerebellum, pons and oblongata	136
Total	1,193

The specimen continued to lose weight slightly and in April, 1908, after removal of the cerebral pia-arachnoid, weighed as follows:

	Grams.
Left hemicerebrum	493
Right hemicerebrum	490
Cerebellum, pons and oblongata	130
Total	1,113

Various calculations indicate that the fresh weight of this brain was probably between 1,200 and 1,250 grams.

The brain is broad and short.[2] The frontal lobes are less massive than in whites. The fissuration is well marked but not very complex. The precallosal length is less than in whites. The callosum is of good size, comparing well with those of whites. The calcarine fissure is interrupted on the left side. The fissural peculiarities must be considered more fully in the final publication, and, if possible, should be based upon comparison with more specimens from natives of this race. The following dimensions may be recorded here:

	Centesimals.
Brain-length, left half	16.1
Brain-length, right half	15.8
Brain-width	13.9
Cerebral index	86.9
Horizontal circumference	47.0
Width, left hemicerebrum	6.9
Width, right hemicerebrum	7.0
Left occipito-temporal length	12.6
Right occipito-temporal length	12.4
Length of callosum	7.3
Percentage of callosal length	45.3%
Left centro-temporal height	10.6
Right centro-temporal height	10.6
Left centro-olfactory height	8.7
Right centro-olfactory height	8.7

Arc Measures.

Left	Frontal	14.5
	Parietal	5.0
	Occipital	5.5
Right	Frontal	14.5
	Parietal	4.5
	Occipital	6.0

Cerebral Indices.

Left	Frontal	58.0
	Parietal	20.0
	Occipital	22.0
Right	Frontal	58.0
	Parietal	18.0
	Occipital	24.0

[2] It was somewhat flattened upon its dorsum during transportation.

HORIZONTAL DISTANCES (IN CENTESIMALS).
FROM FRONTAL POINT TO:

Left Lateral Aspect	1. Tip of temporal lobe	23.0
	2. Sylvian-presylvian junction	31.0
	3. Ventral end of central fissure	44.1
	4. Sylvian-episylvian junction	62.1
Left Mesal Aspect	6. Frontal edge of callosum	22.3
	7. Porta (Foramen of Monro)	42.2
	8. Dorsal end of central fissure	64.6
	9. Dorsal intersection of paracentral fissure	71.4
	10. Caudal edge of callosum	67.9
	11. Occipito-calcarine junction	78.2
	12. Dorsal intersection of occipital fissure	88.2
Right Lateral Aspect	1. Tip of temporal lobe	21.8
	2. Sylvian-presylvian junction	31.4
	3. Ventral end of central fissure	46.8
	4. Sylvian-episylvian junction	63.4
Right Mesal Aspect	6. Frontal edge of callosum	20.5
	7. Porta (Foramen and Monro)	41.6
	8. Dorsal end of central fissure	70.5
	9. Dorsal intersection of paracentral fissure	77.0
	10. Caudal edge of callosum	67.3
	11. Occipito-calcarine junction	76.2
	12. Dorsal intersection of occipital fissure	91.6

Cross-section area of callosum = 5.85 sq. ctm.

NICOBARESE BRAIN.

With regard to the ethnic position of the Nicobarese there exists considerable doubt. They are very different from the Andamanese. Their color is a light brown, the hair is straight and black, and apparently they are of ancient Mongolian origin with probably no admixture of Papuan or Negrito elements. Their stature is medium (158–163 cm.), not small as are the Andamanese.

The brain is that of an individual from Kar Nicobar, a male, aged 25, who died of hypertrophic cirrhosis of the liver and fatty degeneration of the heart in the hospital at Port Blair. The brain weighed 49 ounces avoirdupois or 1,389 grams. The body-weight was 136.5 pounds, while the stature was 170 cm.

Major Anderson injected about 10 c.c. of 5 per cent. formaldehyde into the ventricles through the tuber and immersed the brain in the same mixture. The specimen reached me in March, 1906, a little over three months after its removal from the head.

Its present weight, divested of the cerebral pia-arachnoid, is 1,257 grams. The brain is somewhat flattened and elongated. The fissural pattern is fairly good but not as complex as in the average

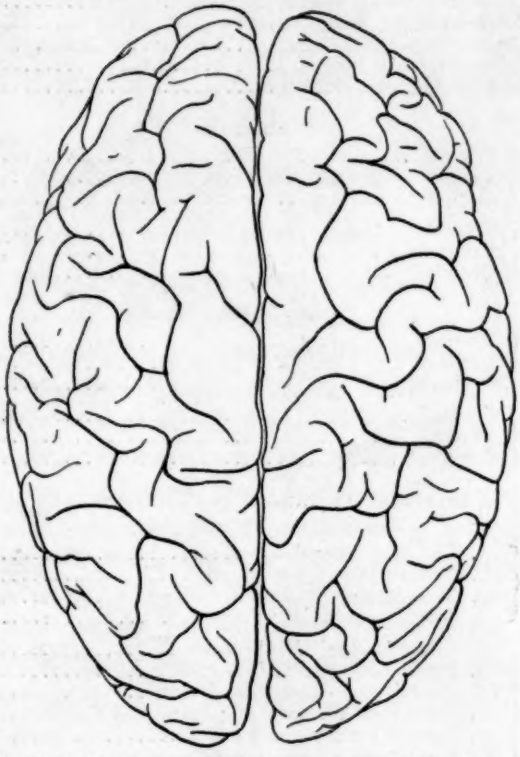

FIG. 2. Brain of Nicobarese (dorsal view).

white brain. The calcarine fissure is interrupted on both sides, the interruption being somewhat concealed on the left side. The callosum is small, a fraction over 5 sq. cm. The indusium, however, is quite massive, and further study of rhinencephalic parts may prove interesting. The insula is slightly visible on both sides.

The measurements of this specimen are as follows:

	CENTESIMALS.
Brain-length, left	19.1
Brain-length, right	18.5
Brain-width	13.3
Cerebral index	70.0
Horizontal circumference	52.0
Width, left hemicerebrum	6.9
Width, right hemicerebrum	6.4
Left occipito-temporal lobe	14.3
Right occipito-temporal lobe	14.2
Length of callosum	8.5

Percentage of callosal length	44.7%
Left centro-temporal height	9.2
Right centro-temporal height	8.7
Left centro-olfactory height	8.2
Right centro-olfactory height	8.0

ARC MEASURES.

Left	Frontal	16.0
	Parietal	4.5
	Occipital	5.5
Right	Frontal	16.5
	Parietal	5.0
	Occipital	5.0

CEREBRAL INDICES.

Left	Frontal	61.5
	Parietal	17.3
	Occipital	21.2
Right	Frontal	62.3
	Parietal	18.8
	Occipital	18.8

HORIZONTAL DISTANCES (IN CENTESIMALS).

FROM FRONTAL POINT TO:

Left Lateral Aspect	1. Tip of temporal lobe	25.1
	2. Sylvian-presylvian junction	30.3
	3. Ventral end of central fissure	41.8
	4. Sylvian-episylvian junction	?
Left Mesal Aspect	6. Frontal edge of callosum	20.0
	7. Porta (Foramen of Monro)	38.2
	8. Dorsal end of central fissure	—
	9. Dorsal intersection of paracentral fissure	65.4
	10. Caudal edge of callosum	64.4
	11. Occipito-calcarine junction	81.1
	12. Dorsal intersection of occipital fissure	83.2
Right Lateral Aspect	1. Tip of temporal lobe	24.6
	2. Sylvian-presylvian junction	28.2
	3. Ventral end of central fissure	41.3
	4. Sylvian episylvian junction	55.0
Right Mesal Aspect	6. Frontal edge of callosum	18.3
	7. Porta (Foramen of Monro)	37.7
	8. Dorsal end of central fissure	61.2
	9. Dorsal intersection of paracentral fissure	63.8
	10. Caudal edge of callosum	63.8
	11. Occipito-calcarine junction	75.4
	12. Dorsal intersection of occipital fissure	87.4

Cross-section area of callosum = 5.02 sq. ctm.

JEFFERSON MEDICAL COLLEGE,
PHILADELPHIA.

DETERMINATION OF DOMINANCE IN MENDELIAN INHERITANCE.

By CHARLES B. DAVENPORT, Ph.D.

(*Read April 25, 1908.*)

The longer one investigates the phenomena of heredity the more one is impressed with the grandeur of the discovery made over forty years ago by Gregor Mendel. His method is not less important than its results. Following him, in studying heredity one considers a single character at a time. One notes the result in the offspring when this character assumes contrasted forms in the two parents or when one parent has the character and the other lacks it. Under these circumstances one frequently, nay, usually, finds that the condition in one parent dominates over that in the other parent, so that the offspring are all alike, and like one parent, in respect to that character. The opposite, or recessive, quality is not lost, however. It persists in the germ plasm and one half of the germ cells of the individuals belonging to the first generation of hybrids contain the dominant and one half the recessive quality.

Dominance, it will be observed, it a matter of the soma. The hybrid fertilized egg contains both contrasting qualities and so, probably, do all of the cells of the body. But only one of the qualities ordinarily makes its appearance. It has been suggested that a struggle occurs between the contrasted qualities and the stronger—called the *dominant*—wins. The question is what determines this assumed greater strength of the dominant quality? What determines dominance?

Various replies have been given to this question. It has been suggested that the dominant quality is the *older* and although this is sometimes true it so often fails to be so that age cannot be regarded as the primary cause of dominance. Frizzling and silkiness of fowl's feathers are each novelties but one dominates over the ordinary flat feather and the other is dominated by it. Much evi-

dence of this sort could be adduced proving the insufficiency of the theory of the recessive nature of novelties. A different theory has been suggested by deVries, namely, when an individual having the characteristic patent is crossed with one in which it is latent the patent characteristic is dominant, the latent recessive. A similar expression has been proposed by Hurst who concludes that the presence of a quality usually dominates over its absence. This expression of the facts is, in the main, true but it is too narrow, inasmuch as it assures that the mendelian result occurs only when a character is crossed with its absence; but this I shall show directly is by no means true.

Two years ago I suggested that a *progressive* variation, one which means a further stage in ontogeny, will dominate over a condition due to an abbreviation of the ontogenetic process—or a condition less highly developed than the first. Recent studies have thrown

FIG. 1.

additional light on this matter and I wish to treat it now generally. First let me present some illustrations. Many poultry have feathers on the feet; these constitute the so-called *boot*. If a "booted" bird be mated with a non-booted all offspring are booted—booting is dominant over its absence. Booting occurs, however, in an infinity of grades. For convenience I recognize ten, usually determined by inspection. If a bird with a boot of grade 8 or 9 be crossed with a bird with boot of grade 2 or 3, both being pure dominants, then the stronger condition is dominant in the offspring, so that their average grade is about 8.

A second illustration may be drawn from certain studies made on the asparagus beetle by Dr. F. E. Lutz, of the Carnegie Institution of Washington. In the embryonic condition the outer wing covers of this beetle are nearly pigmentless or yellow. Before

emerging from the pupal condition black pigment is laid down. The pigmented area is variable in amount. The more extensively pigmented condition is dominant over the less extensively pigmented (*a* over *c*, *d* or *e*—see Fig. 1). In this case, also, it is clear that the facts are better expressed by the statement that the more developed condition dominates over the less developed.

Still another case is that of human eye color. The pigmentation of the iris is variable in amount. The blue iris is without pigment. A small amount of black pigment (with or without yellow) produces the grays; still more pigment yields browns and blacks. Now it appears that the offspring of parents one of whom has gray eyes and the other blue eyes will have gray eyes or blue eyes, but not brown eyes; and gray will show itself dominant over blue. Similarly brown iris color is dominant over gray; the more advanced condition of pigmentation over the less advanced. We have not here to do with a qualitative difference of the presence of a character opposed to its absence, but of a qualitative difference only.

The heredity of human hair color follows a similar law. In one series red pigment is absent in the hair and such colors as flaxen or tow, light brown, brown, dark brown and black may be distinguished. The records collected by Mrs. Davenport and myself show that two flaxen-haired parents have flaxen-haired children and probably only such. Two parents with light brown hair have chil- apparently only such. Two parents with light brown hair have children of two parents each with dark brown or black hair produce children with all of the varieties of hair color. This result means that any lighter color is recessive to any darker color.

The facts recited above and many others thus support the view that, where various stages, *a*, *b*, *c*, in the progressive development of a quality are found in individuals of the same race or species, the more progressive condition will often behave as a dominant toward the less progressive condition. The extreme case is, of course, that in which the organ or quality is absent in one parent and present in the other; but this seems to be only a special case of a more general law.

As to the universality of this law it is still early to speak with confidence. We know too little of the developmental factors of an

organ to decide, in many cases, whether a difference is due to a progressive or a retrogressive change. For instance, the long angora coat of rabbits is recessive to short coat; and this has been cited as a clear case of recessiveness of the advanced condition. But it seems doubtful if such is the case. For the angora coat retains an embryonic quality (viz., of continued growth) which is present in the infancy of the short-haired rabbit and is then inhibited. The inhibiting factor is present in short-haired rabbits and absent in angora rabbits and the presence of the inhibiting factor dominates over its absence. At one time I thought that the dominant white plumage of some poultry was a case of dominance of absence of color. But it now appears that we have among poultry recessive whites which are true albinos, and the *dominant* whites which must be regarded as "grays," in which pigmentation is obscured by an additional factor like that which turns black hair gray. This graying factor is dominant over its absence.

It is possible that the future may show that, in accordance with the ideas of deVries, an advanced grade of a character may be regarded as a sum of minute equivalent elementary units; by the dropping out of these units one at a time a character passes through a series of degradational stages. Then a light brown hair may have one unit of melanic pigment, brown hair two units, dark brown three units, and black hair four units. If this should prove to be true then the four unit condition would dominate over the three unit condition, or the fourth unit would dominate over its absence. But such evidence as I have at present does not favor this view. I am inclined rather to the hypothesis that when the germinal determiner of greater intensity meets that of less intensity it dominates over the latter. This hypothesis receives support from another set of facts which go to prove that the idea of varying intensity of a determiner is a true one. This set of facts is derived from the combs of poultry. In one race of poultry—Polish fowl—the comb consists of a pair of horns or broad flaps which lie far back near the base of the beak; and there is no median comb. In the Minorca and most other fowl there is a single median comb. Now when these two races are crossed we find that the median comb dominates over the absence of median comb; sometimes *completely*, running in the

hybrid from the base of the beak to in front of the nostrils; sometimes *incompletely,* occupying only the anterior half or fourth of the beak. It seems to me clear that in the varying proportions of this median comb in the hybrids we have at once evidence for, and a measure of, varying intensity of dominance. Now it may reasonably be asked whether, when the long-combed and short-combed hybrids are mated together, the long comb dominates over the short. The answer is complicated by the fact that the Polish "horns" reappear in this second generation; but, leaving this aside, we find that there is a greater preponderance of *long* median combs than simple mendelian expectation calls for and this indicates that the longer median comb tends, but not always perfectly, to dominate the shorter median comb; or, in other words, the more intense determiner dominates the less intense.

To sum up, I think it is clear that dominance in heredity appears when a stronger determiner meets a weaker determiner in the germ. The extreme case is that in which the strong determiner meets a determiner so weak as to be practically absent as when a red flower is crossed with a white. In such cases we have the clearest examples of mendelian inheritance. But there is an entire gamut of cases where the opposed determiners are of varying relative potency. The phenomenon of determinance is seen in these cases also; but the mendelian law in them is sometimes obscured and sometimes merely not applicable.

COLD SPRING HARBOR, LONG ISLAND, N. Y., April, 1908.

PROC. AMER. PHIL. SOC. XLVII. 188 E, PRINTED JULY 10, 1908.

THE SANTA CRUZ TYPOTHERIA.

(Figures 1–10.)

By WILLIAM J. SINCLAIR.

(*Read April 24, 1908.*)

The Typotheria are a group of semi-ungulate mammals of strictly South American origin appearing first in the Notostylops beds of Patagonia.[1] During the Santa Cruz epoch four genera are represented but what is lacking in generic and specific diversity is more than compensated for by an abundance of individuals. The total number of common species apparently does not exceed eight, but this

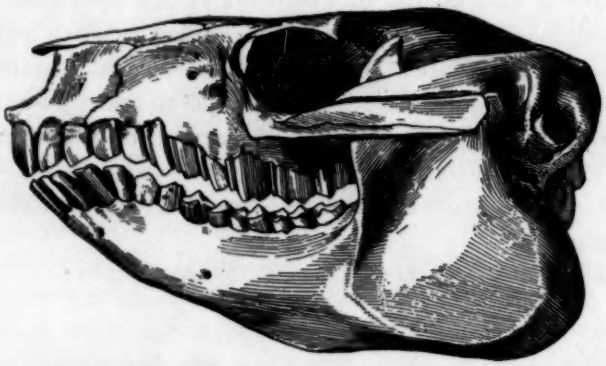

FIG. 1. Skull of *Protypotherium australe* Ameghino, side view, three fourths the natural size. (No. 9565 American Museum of Natural History collection.)

has been increased to no less than fifty-one by failing to estimate at their true value characters due to age and others which seem to be of the nature of individual variations in size, the result no doubt of the extremely fragmentary character of the material hitherto available. Even with the large suites of specimens in the collections

[1] *Isotypotherium, Epitypotherium.*

at Princeton University and the American Museum of Natural History it has been found impossible to separate in a satisfactory manner the species of the genus *Protypotherium*. An almost exact intergradation in size without appreciable difference in structure is observable between the largest species *Protypotherium australe* (Fig. 1) and the smallest *P. attenuatum*. As none of the collections have been made with strict regard to stratigraphic sequence, we are not in a position to say whether these differences represent individual variations or true mutations. The former alternative has been adopted in monographing the group (see the forthcoming Volume VI., Part I. of the "Reports of the Princeton University Expeditions to Patagonia").

Classification of the Santa Cruz Typotheria.

The Typotheria are grouped by Scott[2] as a suborder of the Toxodontia and may be defined as follows:

Plantigrade or digitigrade mammals with pentadactyl[3] or tetradactyl feet, strongly interlocking carpus with os centrale and serial or slightly interlocking tarsus with hemispherical astragalar head. Dentition usually complete but tending toward reduction of the lateral incisors, canine and anterior premolars in specialized forms. Median incisors more or less enlarged and functional as cropping teeth. Molars hypsodont, lophoselenodont in crown pattern, curving inward above and outward below. A clavicle is present in some forms. Femur with third trochanter. Fibula articulating with calcaneum.

Two well-marked families are recognizable among the Santa Cruz representatives of the suborder for which the names Interatheridæ and Hegetotheridæ have priority. Each contains a large and a small genus of which, in either case, the former is the less specialized. The following key to the families and genera may facilitate the determination of new material:

[2] Scott, W. B., "The Miocene Ungulata of Patagonia," *Rept. British Asso. Adv. Sci.*, 1904, pp. 589–590.

[3] Ameghino figures a pentadactyl manus in *Pachyrukhos typicus*, "Contrib. al conocimiento de los mamíferos fósiles de la República Argentina," *Actas de la Academia Nacional de Ciencias en Córdoba*, T. V., Pl. 13, fig. 14, 1889, and in *Typotherium, ibid.*, Pl. 18, fig. 5.

Order *TOXODONTIA* Owen.

Suborder TYPOTHERIA Zittel.

A. Family INTERATHERIDÆ. Median incisors rooted; third and fourth premolars not completely molariform, squamoso-mastoid region dilated and cancellous; malar long and narrow, inclosed between temporal process of maxillary and squamosal; maxillary orbital; carotid canal and foramen lacerum posterius fused; tibia and fibula unfused distally; pes paraxonic, digits II. and V. equally reduced and small, digits III. and IV. large and of equal length; astragalar trochlea bilaterally symmetrical; no naviculo-calcaneal facet; calcaneum with large fibular facet.

1. *Protypotherium.* Dental formula $\frac{3}{3}, \frac{1}{1}, \frac{4}{4}, \frac{3}{3}$ in close series. Lateral incisors unreduced; canine incisiform; upper molars with deep internal inflection and slight antero-external ridges; M^3 externally bilobate; temporal bar of maxillary with slight descending process; humerus with internal epicondylar foramen; terminal phalanges laterally compressed hoofs with slight clefts in manus.

2. *Interatherium.* Dental formula $\frac{3}{3}, \frac{1}{1}, \frac{4}{4}, \frac{3}{3}$, with diastemata between the lateral incisor, canine and first premolar, varying with the species. I^3 reduced, often wanting; upper molars with deep internal inflection and prominent antero-external ridges; M^3 externally trilobate; temporal bar of maxillary with strong descending process; humerus without internal epicondylar foramen; terminal phalanges laterally compressed hoofs with or without clefts.

B. Family HEGETOTHERIDÆ. Median incisors rootless; third and fourth premolars molariform; mastoid dilated inclosing a large hollow cavity; malar large excluding maxillary from orbit; carotid canal and foramen lacerum posterius widely separated; tibia and fibula firmly fused both proximally and distally; pes approaching mesaxonic with digit III. the longest, digit V. greatly reduced and digits II. and IV. shorter than III. but robust; astragalar trochlea bilaterally asymmetrical; navicular and calcaneum in articulation; small fibulo-calcaneal facet.

1. *Hegetotherium. Dental formula* $\frac{1}{1}, \frac{1}{1}, \frac{4}{4}, \frac{3}{3}$. Second and third upper and third lower incisor vestigial; canine vestigial; upper molars internally convex, without inflection except in M^4; ectoloph smooth; terminal phalanges greatly flattened transversely with prominent clefts.

2. *Pachyrukhos.* Dental formula $\frac{1}{1}, \frac{0}{0}, \frac{4}{4}, \frac{3}{3}$. All the upper molars internally convex; ectoloph smooth; terminal phalanges hoof-like without clefts in Santa Cruz species.

The Santa Cruz typotheres are animals of somewhat rodent-like appearance, varying in size from a cotton-tail rabbit to a cavy. A review of the more important skeletal characters of the group may be of value, even though it involve some repetition.

FIG. 2. Skull of *Interatherium robustum* Ameghino, side view, three fourths the natural size. (No. 9263 American Museum of Natural History collection.)

1. *The Skull.*—The facial portion of the skull is slender and more or less excavated longitudinally while the brain case is broad and well expanded. The orbits are central, circular in outline, quite prominent in *Hegetotherium, Pachyrukhos* and *Interatherium* and unenclosed posteriorly. The jugal arches are robust in all except

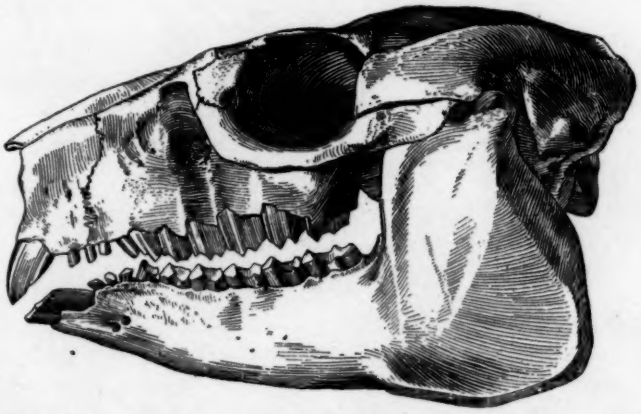

FIG. 3. Skull of *Hegetotherium mirabile* Ameghino, side view, three fourths the natural size. (No. 15542 Princeton University collection.)

Pachyrukhos and moderately expanded. The premaxillæ are short and heavy with scarcely any ascending process; the nasals are broad posteriorly, tapering forward to blunt points; the interorbital tract plane and the sagittal and lambdoidal crests low. The most promi-

nent feature of the back of the skull is the greatly distended mastoid tract which may either be filled with cancellæ or lodge a large cavity. In either case there is direct communication with the tympanic bulla and the dilation appears to have functioned as a secondary resonator, perhaps associated with nocturnal habits. The palate is concave throughout, terminating posteriorly in a pair of stout processes. The mandible is heavy and deep, without trace of suture in the firmly fused symphysis.

2. *Dentition.*—Beginning with the normal incisor formula in *Protypotherium* (Fig. 1) the Santa Cruz typotheres show a well-marked tendency toward an increase in size of the median incisors at the expense of the lateral incisors, canine and anterior premolar until the extreme stage of reduction in *Pachyrukhos* (Fig. 4) is attained. The teeth undergoing elimination are reduced to simple cylinders. It is not to be understood that *Protypotherium, Interatherium, Hegetotherium* and *Pachyrukhos* constitute a phyletic series because they represent successive stages in the process of dental reduction associated with the hypertrophy of the median incisors. As already indicated in the key to the genera, two divergent lines are represented and not a single progressive series. A rather curious feature of the lower incisors in *Protypotherium* is the presence in the first and second of a deep median cleft producing a fork-like structure recalling a somewhat similar division of the lower incisor crowns in the Hyracoidea. In all the Santa Cruz typotheres the enamel layer on the enlarged incisors tends to be confined to the anterior surface of the crown. The molars in all the genera are constructed on much the same plan but only in *Protypotherium* are absolutely unworn teeth known, consisting essentially of a broadly concave ectoloph (*e*, Fig. 8, *A*) and a pair of crescents with the convexity directed inward (*ac, pc,* Fig. 8, *A*), of which the anterior horns are fused with the ectoloph inclosing a reentrant. A crista-like ridge from the ectoloph (*c,* Fig. 8, *A*) is separated from the anterior crescent by a deep notch. A slight ridge (*pp,* Fig. 8, *A*) blocks the shallow valley inclosed by the posterior crescent. As the tooth wears the antero-external angle of the crown elongates and is channeled by a shallow groove producing the ridges noted in the key to the genera.

In the lower molars the convexity of the crescents is reversed so that the reentrant fold is external (Figs. 1, 3, 9, *A*). A prominent lobe spanning the arc of the posterior crescent (*pp*, Fig. 9, *A*) is not peculiar to the teeth of the Typotheria alone, but is present also in *Nesodon* (Fig. 9, *B*), *Astrapotherium*, *Theosodon* and other extinct ungulates from South America. In the last lower molar the development of the third lobe present in *Interatherium* is accomplished by the deepening of the shallow groove indicated in *Protypotherium* at the point marked *pc* in Fig. 9, *A*.

As mentioned in the generic key the premolars are sometimes molariform and sometimes not, differing from the molars in the latter case in having the anterior crescentic lobe smaller than the posterior.

Roots are developed only in the deciduous molars but as these have been observed only in *Protypotherium* and *Interatherium* it is

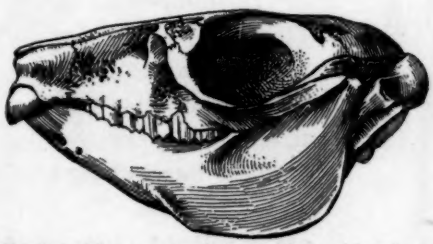

FIG. 4. Skull of *Pachyrukhos moyani* Ameghino, side view, three fourths the natural size. (Reconstructed from several specimens.)

not altogether certain whether this character is of family or subordinal value. So far as can be ascertained the crown pattern seems to have been the same in the deciduous and permanent series, the milk teeth resembling their successors. The order of replacement seems to have been the normal one.

A thin layer of cement is usually observable on the molars and premolars of all the genera.

3. *Axial Skeleton.*—The dorso-lumbar vertebral formula in *Interatherium* is twenty-two, of which fifteen are dorsals. It was probably the same in *Protypotherium* but in *Pachyrukhos* eight lumbars are present. Five vertebræ are coössified, in the sacral complex of which three are true sacrals in contact with the ilium and

two belong to the caudal series. The length of the tail seems to have varied. In *Protypotherium* and *Interatherium* it is both long and heavy while in *Pachyrukhos* there is reason to believe that it was quite short.

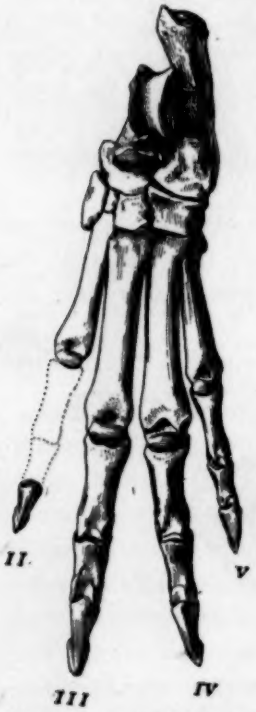

FIG. 5. Left hind foot of *Protypotherium australe* Ameghino, three fourths the natural size. (No. 9149 American Museum of Natural History collection.)

4. *Foot Structure.*—Almost nothing has hitherto been known of the structure of the feet in the Santa Cruz typotheres, but definite information is now available for all the genera except *Hegetotherium,* in which the manus is still unknown, but from the close structural resemblance of *Hegetotherium* and *Pachryukhos* it is probable that it was not unlike that of the latter, which in turn does not differ materially from the manus of *Interatherium* and *Protypotherium* (Fig. 6, *A*). In the Santa Cruz forms both manus and pes are tetradactyl without the slightest trace of an opposable thumb or great toe.[4] The carpus is strongly interlocking and shows no trace of the centrale. Two types of hind foot are developed (Figs. 5 and 7, *A*) simulating the paraxonic and mesaxonic symmetry of the feet of the Artiodactyla and Perissodactyla. These are probably to be correlated in the Typotheria with cursorial and saltatorial modes of progression. *Pachyrukhos* was certainly a jumping animal as shown by the greater length and strength of the hind limbs and inner digits of the pes. In fact, the structure of both the fore and hind limbs in this animal

[4] A pentadactyl manus with separate centrale in the carpus and opposable thumb and a pentadactyl pes with large opposable hallux figured by Ameghino, *Revista Argentina de Hist. Nat.,* I., pp. 393, 394, figs. 95, 96 and referred to *Interatherium* (*Icochilus*) *robustum* do not pertain to this genus. The same figures with the erroneous determination appear also in Zittel's "Handbuch der Palaeontologie," IV., p. 493, fig. 407.

closely resembles that of the rabbit. From the numerous structural similarities between *Pachyrukhos* and *Hegetherium* it may be inferred that the latter was also saltatorial. Its broad, shallow astragalar trochlea is in contrast with the narrow, more deeply incised trochlea of the cursorial *Protypotherium* and *Interatherium*. Both of these genera have the fore and hind limbs of approximately equal length. The terminal phalanges in the Santa Cruz typotheres are hoof-like and in *Hegetotherium* have prominent median clefts.

Relationships of the Santa Cruz Typotheria.

1. *With the Toxodonta.*—In the evolution of the teeth and feet, the Santa Cruz Typotheria are less advanced than their contemporaries, the Nesodons. The feet of *Nesodon* (Figs. 6, *B*, 7, *B*) are

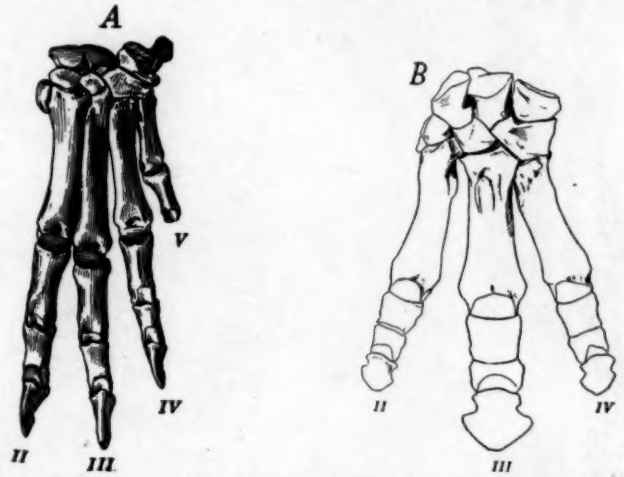

Fig. 6. *A*. Left fore foot of *Protypotherium australe* Ameghino, three fourths the natural size. (No. 9149 American Museum of Natural History collection.) *B*. Left fore foot of *Nesodon imbricatus* Owen, about one fifth the natural size. (No. 15460 Princeton University collection.)

tridactyle with the axis passing through the third digit. The manus has originally been tetradactyl like that of *Protypotherium* (Fig. 6, *A*) but has lost almost all trace of the fifth digit, a mere vestige, not shown in the figure, remaining. The other bones of the wrist and foot have not suffered any displacement as a result of this loss but interlock in the same way as in *Protypotherium*. The hind foot

of *Nesodon* (Fig. 7, *B*) is the realization of a structure already foreshadowed in the pes of *Hegetotherium* (Fig. 7, *A*). The fifth digit, which is greatly reduced in *Hegetotherium*, has here disappeared and the ento- and meso-cuneiforms have united to a single bone. The shortening of the neck of the astragalus and the increase in size of the fibular facet on the calcaneum are, perhaps, adaptations to the support of weight. Although the molars of *Nesodon* appear

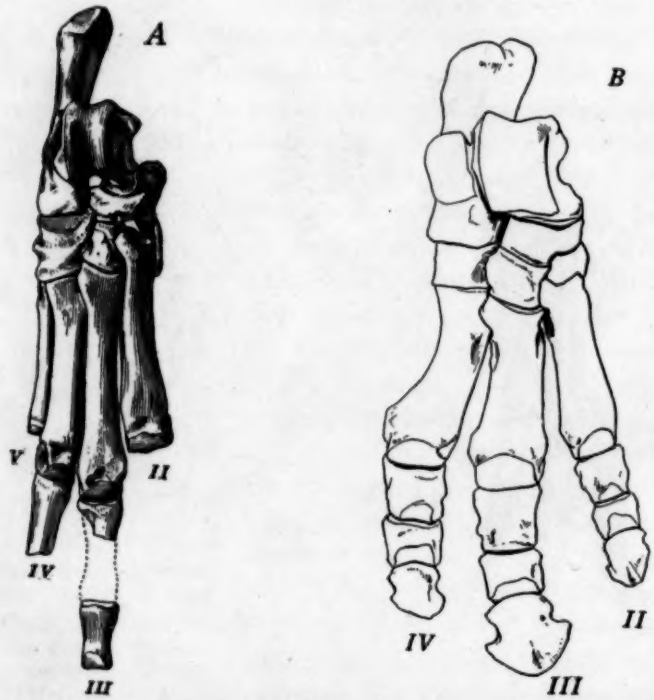

FIG. 7. *A. Hegetotherium mirabile* Ameghino, right hind foot, three fourths the natural size. (No. 15542 Princeton University collection.) *B. Nesodon imbricatus* Owen, right hind foot, about one third the natural size. (No. 15460 Princeton University collection.)

exceedingly complex, owing to the development of secondary enamel folds, the primary elements can be homologised with those displayed in the simpler crown pattern of *Protypotherium*, as indicated by the similar lettering in Figs. 8 and 9. This comparison can not yet be

extended to the other Santa Cruz genera, *Hegetotherium, Pachyrukhos* and *Interatherium,* as unworn molars of these are not available. *Nesodon* differs from the Typotheria in the enlargement and caniniform character of the second incisor above and the third below, while in the Typotheria the median incisor in both jaws is the only one tending toward great increase in size. In none of the Santa Cruz Typotheria is there a trace of the double deciduous dentition characteristic of *Nesodon.*

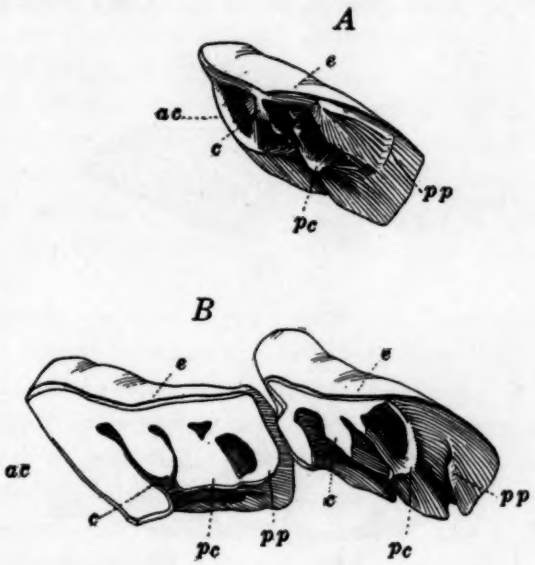

FIG. 8. *A.* Unworn third upper molar of a young *Protypotherium,* four and one half times the natural size. (No. 9482 American Museum of Natural History collection.) *B. Nesodon imbricatus* Owen, second and third upper molars slightly worn, three fourths the natural size. (No. 15135 Princeton University collection.) *ac,* antero-internal crescent; *pc,* postero-internal crescent; *e,* ectoloph; *c,* crista; *pp,* posterior pillar.

From these resemblances in dentition and foot structure it seems permissible to infer that the Toxodonta and Typotheria had a common origin, but the facts at present available do not justify us in saying more.

2. *With Typotherium.*—Difficult as it is to ascertain the relationship existing between the Santa Cruz Typotheria and the Nesodons,

it is even more so to determine their degree of kinship with *Typotherium*. From their small size it seems quite probable that none of the Santa Cruz Typotheria are in the direct line of descent culminating in this genus. This is confirmed by the degree of specialization in dentition and foot structure which *Typotherium* displays. The teeth of the latter show a greater complexity of folding than is attained by any of the Santa Cruz typotheres, while the feet are less specialized with a pollex in the manus which has been lost in *Protypotherium*, the most generalized of the Santa Cruz typotheres

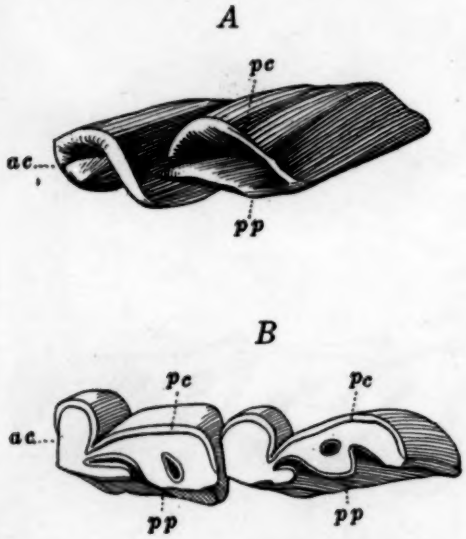

FIG. 9. *A*. Unworn third lower molar of a young *Protypotherium*, four and one half times the natural size. (No. 9482 American Museum of Natural History collection.) *B. Nesodon imbricatus* Owen, two lower molars, three fourths the natural size. (No. 15135 Princeton University collection.) *ac*, anterior crescent; *pc*, posterior crescent; *pp*, posterior pillar.

(Fig. 6, *A*), and with digit V. of the pes less reduced than in the most specialized of the latter (*Pachyrukhos*). A pollex has been figured by Ameghino[5] in the manus of *Pachyrukhos typicus*, but none has been found in any Santa Cruz specimen. The manus in *Hegetotherium* is unknown, so the above statement regarding the

[5] Ameghino, Florentino, "Contrib. al conoc., etc.," Pl. 13, fig. 14.

degree of specialization in foot structure displayed by *Typotherium* may require some modification in the light of fuller knowledge.

3. *With the Rodents.*—In many features of skull and skeleton the Typotheria resemble the rodents. This is most apparent in *Pachyrukhos,* which seems to have been a saltatorial animal, but in none of the Typotheria are the following characters peculiar to rodents developed:

A. Persistently growing, chisel-shaped incisors (I. ⅔ of the permanent series, Weber).[6] I. ⅓ of the permanent series is enlarged in some of the Typotheria and may grow persistently but is modified for cropping and not for gnawing.

B. More or less antero-posterior elongation of the mandibular condyle and corresponding modification of the glenoid fossa to permit backward and forward movement of the lower jaw. In the Typotheria the condyle is approximately circular in outline with the glenoid surface flattened and the movement of the mandible is from side to side.

C. Frequent outward curvature of the crowns of the upper molars and inward curvature of those of the inferior series in hypsodont forms. The reverse is true in the Typotheria.

D. Contact of ascending process of premaxillary with frontal. This process is short and robust in the Typotheria and is widely separated from the frontal by the maxillary.

E. Elongation of the mandibular angle. The angle is evenly convex in the Typotheria.

F. The astragalus in rodents is characterized by a broad, short, rather shallow trochlea with the crests sharp and equally developed, distinct neck and flattened head, convex distally; trochlea symmetrical to the vertical plane; fibular and internal malleolar facets vertical; body limited posteriorly; no astragalar foramen. In the Santa Cruz Typotheria the body is deeper than in rodents, the crests may or may not be equally developed and the head is globular without antero-posterior flattening. The symmetry of the trochlea with respect to the vertical plane varies in the different families. In the other characters they resemble rodents.

[6] "Die Säugetiere," p. 480, 1904.

G. The presence of a free centrale in the carpus in all rodents except the Hystricidæ and *Caelogenys* and the general fusion of the scaphoid and lunar in all except the Bathyergidæ, Ctenodoctilidæ and Lagomorpha.[7] The centrale is wanting in the carpus of the Typotheria and the lunar is always free.

H. The presence of a tibial sesamoid in all the simplicidentate rodents. This is not found in the tarsus of the Typotheria.

The Typotheria resemble rodents in the elongation of the anterior portion of the skull with the reduction of the incisor-canine-premolar series (*cf.* Figs. 3 and 4), in the enlargement and often permanent growth of the median incisors (not homologous with the enlarged incisors in rodents, see under *A*, above), in the development of a mastoid dilation which may be filled with cancellæ (Interatheridæ) as in many rodents and connected with the auditory bulla, in the shape of the proximal articular surfaces between the radius and ulna, in the broad anteriorly directed transverse processes of the lumbar vertebræ and in several other characters of minor importance.

FIG. 10. Left hind foot of *Procavia* (*Dendrohyrax*) *arborea*, ¾ natural size. (No. 365 Princeton University osteological collection.)

In view of the striking differences in structure indicated in the preceding paragraphs, it seems probable that these resemblances are to be explained as instances of convergence.

4. *With the Hyracoidea.*—A more or less intimate relationship between the Typotheria and Hyracoidea is commonly assumed but with the complete material now available it is difficult to see on what grounds this hypothesis can be maintained. The hyracoid carpus is arranged on the linear plan with separate centrale while in the tarsus the astralagus is unlike that of any other mammal in possessing a large step-like articulation for the internal tibial malleolus (Fig. 10).

In striking contrast with hyrax, the carpus in the Typotheria is strongly interlocking without centrale, and the internal tibial malleolus is applied to the lateral surface of the astragalus without trace

[7] Weber, *loc. cit.*, p. 476.

of the supporting shelf (Figs. 5 and 7, *A*). The flat astragalar head in the Hyracoidea and the articulation of the fibula with the astragalus instead of with the calcaneum are additional points of difference, all of which are more than sufficient to offset similarities in skull structure which are confined to a few points, such as the cancellous dilation of the mastoid, the shape of the posterior border of the palate, and the increase in depth posteriorly of the mandible. In the Hyracoidea the molar takes part in forming the outer portion of the glenoid cavity, the parietal enters into the postorbital process and the base of the coronoid just back of the last lower molar is perforated by a large foramen, a superior branch of the alveolar canal. None of these characters are exhibited by the Typotheria. In the hyracoid dentition, the first upper incisor is a persistently growing downwardly curved tusk of triangular cross-section. In some of the Typotheria this tooth may grow persistently but it is always antero-posteriorly compressed, transversely expanded and modified for cropping, never appearing as a tusk. The molars of the Hyracoidea are lophoselenodont and either brachyodont or short hypsodont while in the Typotheria they are extremely hypsodont, developing roots only in the deciduous series. The crown pattern of the hyracoidean molar bears more resemblance to that of some of the early horses and rhinoceroses than to the molar pattern in the least specialized of the typotheres (Figs. 8 and 9).

The so-called hyracoids from the Fayum Province of Egypt (*Saghatherium, Megalohyrax*) are as yet known only from fragments of the skull and dentition but, so far as the available material permits comparison, resemble the modern Hyracoidea and not the Typotheria which would probably not be the case if the two orders were related as it would naturally be expected that a closer similarity should exist between the Eocene and Miocene representatives of an order than between the latter and the recent forms. All the Egyptian hyracoids have the base of the coronoid perforated by a branch of the dental canal as in the recent forms[8] and unlike the Typotheria.

[8] Communicated by Mr. Walter Granger, of the American Museum of Natural History.

Various pre-Santa Cruz genera (*Archæohyrax, Argyrohyrax*) have been referred to the Hyracoidea. Their foot structure is still unknown but the skull and dentition, to judge from the photographs, figures and descriptions examined by the writer, are not hyracoidean in character. Too little is known of these forms to warrant a discussion of their relationship with the Santa Cruz Typotheria, but there can be little doubt that they should be referred to the same suborder.

PRINCETON UNIVERSITY, April, 1908.

NOTES ON SOME CHILEAN COPPER MINERALS.

By HARRY F. KELLER.

(*Read April 24, 1908.*)

Some time ago my brother, Mr. Hermann A. Keller, presented me with a fine suite of mineral specimens collected by him on a professional trip to Chilean mining localities. The minerals, which include native sulphur and copper, various oxides, chlorides, sulphates, borates and silicates, were for the most part readily identified by their characteristic appearance or by simple tests, but some of them aroused my curiosity, partly because of their rare occurrence, and partly on account of their beauty or exceptional purity. I was thus led to make a number of qualitative and quantitative analyses, the results of which appear to me sufficiently interesting to be placed on record. In the present paper I shall confine myself to the description of some minerals containing copper as either a principal or a minor constituent.

Cupreous Manganese.

It is well known that in many varieties of psilomelane or wad the manganous oxide is partially replaced by oxide of copper, and that special names have been given to some of those varieties in which the proportion of the latter oxide is considerable. Among them is the peloconite from Remolinos, Chile, which was first described by Richter,[1] and chemically characterized by Kersten.[2] Its quantitative composition, however, does not appear to have been fully determined. The material supplied by my brother included several very fine specimens of a cupreous manganese from Huiquintipa, Province of Tarapacá, and these are unquestionably identical with Richter's peloconite. With the one exception of the specific gravity, the physical and chemical characters of the new material are precisely

[1] *Poggendorffs Annalen*, 21, 590.

[2] *Schweigger's Journal*, 66, 7.

similar to those of the Remolinos occurrence. The mineral is massive and amorphous, has a conchoidal fracture, a bluish-black color and a liver-brown streak. Its hardness is between 3 and 4, and the specific gravity 3.683 (instead of 2.5–2.6). When broken into small pieces and carefully picked with the aid of a lens, the material appeared quite homogeneous except for a few particles of quartz and some green or bluish specks of a copper compound on the outside and along the crevices. A qualitative analysis showed that it contains the oxides of manganese, copper and iron, together with water and varying amounts of admixed silica. To ascertain whether the mineral has a definite chemical composition analyses were made of carefully selected samples from different specimens. It was found that the silica, which separates on dissolving the substance in hydrochloric acid, is not uniformly distributed through the mass. Its percentage varied from 12 per cent. to 32 per cent., and its microscopic examination showed that it consists entirely of quartz. There could be no doubt, then, that it is simply an admixture, and that in calculating the composition, the silica (of which only a trace dissolves with the mineral) should first be deducted from the amount of the substance taken. The results of the analyses were as follows:

	I.	II.	III.	IV.
Oxygen	14.37%	14.18%	13.89%	not det.
Manganous oxide	69.61	68.95	69.44	70.61%
Cupric oxide	5.86	6.05	5.69	6.48
Cobalt oxide	.48	.56	not det.	not det.
Barium oxide	.36	.47	not det.	not det.
Ferric oxide	2.05	1.94	1.89	4.22
Alumina	1.92	1.91	2.10	
Water	5.14	5.29		
	99.79%	99.35%		

It is seen from these figures that the proportions of the several constituents of the mineral are fairly constant. The composition is that of psilomelane, in which part of the manganese is replaced by copper. It is difficult to account for the very constant proportions of oxide of iron and alumina.

Regarding the determinations of water and of available oxygen, I may mention that the former was made by heating the substance

in a current of dry air and collecting the moisture in calcium chloride, while the latter was estimated iodometrically in I. and II., and indirectly in III., by heating a weighed portion, first in air and then in hydrogen, and allowing for the water and the reduction of the oxides of copper and iron. As a matter of course this method is less reliable, but the result nevertheless agrees quite well with the iodometric determinations.

Chalcanthite and a Double Sulphate of Copper and Magnesium.

Among the specimens that claimed my special attention there was one[3] consisting of irregular and rounded masses, and which was labeled "sulphate of copper and aluminium." While the shape of the little lumps was about the same, three distinct kinds of material could readily be picked from the specimen, even without the help of a magnifying glass.

One of these substances had a deep blue color and was recognized without difficulty as chalcanthite. The blue masses were evidently crystals which were strongly corroded and slightly effloresced on the surface. A quantitative analysis confirmed the composition $CuSO_4 + 5H_2O$, with very small amounts of iron and magnesium sulphates, and a slight admixture of silicious matter. It gave:

	Found. Per Cent.	Calculated. Per Cent.
Sulphur trioxide	32.21	32.1
Cupric oxide	31.52	31.8
Ferrous oxide	.30	
Magnesium oxide	.35	
Water	35.79	36.1
	100.19	100.0

More interesting were the bluish-white masses which formed the larger portion of the specimen. They were earthy and friable, but presented shapes and surfaces exactly similar to those of the chalcanthite, suggesting a pseudomorph after the latter. In composition, however, the material was found to differ from chalcanthite in

[3] From Copaquire, Province of Tarapacá.

that it contained a large proportion of magnesium sulphate. Analyses of two different samples yielded:

	Found.		Calculated for $(Cu, Mg)SO_4+5H_2O$:
	I.	II.	
Sulphur trioxide	35.74%	35.67%	35.84%
Cupric oxide	12.41	12.46	11.89
Magnesium oxide	11.42	11.36	11.95
Ferrous oxide	.97	1.05	
Manganous oxide	.23	.41	
Nickel oxide	trace	.06	
Water	38.45	38.31	40.32
	99.22%	99.32%	100.00%

The conclusions to be drawn from these results are, first, that the mineral is an isomorphous mixture of the sulphates of copper and magnesium; secondly, that this double salt contains five molecules of water of crystallization; and thirdly, that for each molecule of copper sulphate there are present (very nearly) two molecules of magnesium sulphate. The shortage in the water content is doubtless owing to efflorescence, and there should be credited to the magnesia content an amount equivalent to the percentages of the oxides of iron and manganese.

Under the name of cupromagnesite a double sulphate of copper and magnesium has been described by Scacchi. It occurs in the form of green crusts on lava from the Vesuvius, and is believed to be isomorphous with melanterite, containing, like the latter, seven molecules of water of crystallization. I have seen no reference to a mineral of the same composition as that above described.

Associated with the chalcanthite and the double sulphate of copper and magnesium were other little masses, dirty-white in color and more or less stained with ferric oxide. They were very hard and consisted almost entirely of silica, containing only trifling amounts of oxide of iron and magnesia. It is puzzling to explain why these masses should simulate the form of the accompanying soluble sulphates.

Brochantite(?) Containing Arsenic Acid.

Very small quantities only were available of an emerald green mineral which was observed partly in fine acicular crystals dissemi-

nated through a silicious rock, and partly as an incrustation upon quartz. On account of its physical characters, as well as the strong reaction its solution gave with barium chloride, I was first inclined to regard it as a typical brochantite. This impression was confirmed by rough estimations of the copper and sulphur trioxide, but as these tests had been made on impure material, I decided to attempt the analysis of a carefully prepared sample. To obtain about .5 grm. of the substance, I found it necessary to sacrifice the best specimens in my possession, and my patience was put to a severe test in picking the minute crystals under the lens. They were sorted over and over until the microscope showed only a few remaining specks of quartz adhering to the larger crystals of the copper mineral.

The quartzy material from which this sample had been picked still contained considerable quantities of the copper mineral, and it occurred to me that it might serve for a qualitative, and, perhaps, a preliminary quantitative analysis. Accordingly the material was extracted with hydrochloric acid, and the resulting green solution divided into equal parts. When the copper had been precipitated with hydrogen sulphide, it was noticed that yellow flakes began to form, and after the liquid saturated with the gas had been allowed to stand in a warm place over night, a considerable amount of the yellow precipitate had settled on the black copper sulphide. There could be no doubt, then, that arsenic was present in the form of arsenic acid. The sulphides were separated and worked up in the usual manner, and the filtrate was searched for other metallic ions. It yielded only traces of iron oxide and alumina. The other half of the original solution was used for the determination of the sulphur trioxide and the arsenic acid. The results of the determinations, calculated for the entire amount of copper mineral dissolved, gave:

Copper	.5068 grms.
Sulphur trioxide	.0783 grms.
Arsenic anhydride	.1309 grms.

The question now arose as to whether the very large proportion of arsenic acid found really constituted an integral part of the supposed brochantite, or whether it did not belong to another mineral contained in the rock. The test previously made seemed to preclude such an arsenic content of the mineral under examination.

In view of the very limited amount of material available for analysis, it seemed best to dispense with the water determination and confine the characterization of the mineral to ascertaining the specific gravity and an estimation of the base and the acids. The following results were obtained:

Specific gravity	3.160	
	I. Per Cent.	II. Per Cent.
Sulphur trioxide	16.32	16.63
Arsenic anhydride	2.31	2.40
Copper oxide	68.90	68.68
Silica	1.63	1.18

If we deduct the quartz from the substance taken for analysis, the percentages of sulphur trioxide and oxide of copper will be found to approach very nearly to those in brochantite which contains:

	Per Cent.
Sulphur trioxide	17.7
Cupric oxide	70.3
Water	12.0

and it is difficult to explain the rôle of the arsenic acid which is equivalent to about 4.4 per cent. of sulphur trioxide. To establish a definite formula for the compound it would be necessary to ascertain by further analyses whether or not the proportion of arsenic anhydride is constant, and to complete the analysis by an exact determination of the water. The specimens in my possession, unfortunately, are not sufficient for this purpose.

The specimens were collected at Copaquire, Province of Tarapacá.

Atacamite.

In conclusion I desire to call attention to some magnificent specimens of atacamite from Paposo in the Province of Antofagasta. They do not show the usual slender prisms, but consist of aggregates of fairly large crystals, closely resembling the octahedron of the isometric system modified by the cube and the rhombic dodecahedron. On closer examination, however, it would seem that they are really combinations such as have been observed on the atacamite

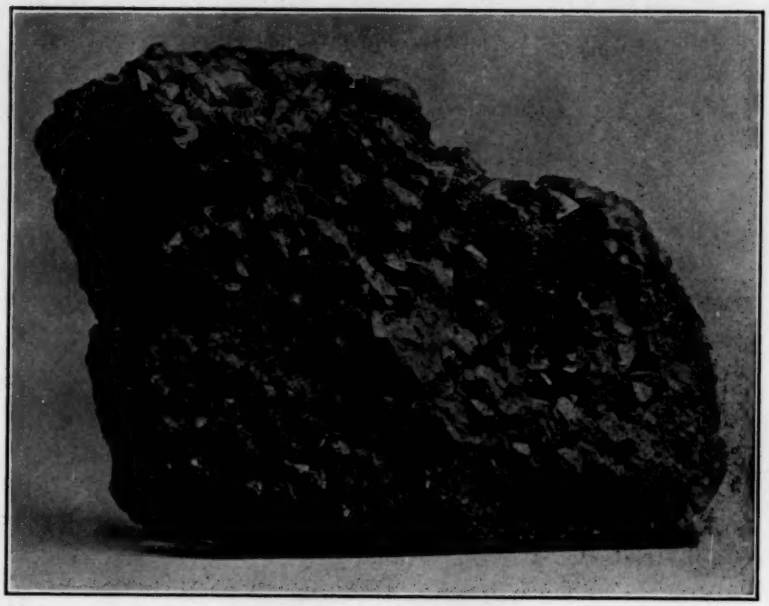

FIG. 1. Atacamite from Paposo, Chile.

from certain localities in South Australia.[4] As was to be expected, the analysis showed the specimens to be an atacamite of unusual purity and of normal composition. It yielded:

Sp. gr. 3.738

	Found. Per Cent.	Calculated for $CuCl_2.3Cu(OH)_2$. Per Cent.
Chlorine	16.51	16.6
Copper	14.79	14.9
Cupric oxide	55.28	55.8
Water	12.42	12.7
Insol.	.79	
	99.79	100.0

CENTRAL HIGH SCHOOL, PHILADELPHIA.

[4] I hope to verify this by actual measurement.

PROGRESS OF THE DEMARCATION OF THE ALASKA BOUNDARY.

By O. H. TITTMANN,

U. S. Commissioner.

(*Read April 24, 1908.*)

The boundary between the British and Russian possessions in North America was defined by the Treaty of St. Petersburg of 1825. When the United States purchased the Russian possessions, or Alaska, in 1867, it was believed that the territorial jurisdiction of the United States and Great Britain could not become a matter of controversy. This view is evidenced by the remark made by Charles Sumner in his speech advocating the purchase of Alaska. "I am glad," said he, "to begin with what is clear and beyond question. I refer to the boundaries fixed by the treaty."

The total length of the boundary referred to by Mr. Sumner is twelve hundred miles. It divides itself naturally into two sections of about six hundred miles each. One is the section bounded by the 141st meridian, and the other the irregular boundary delimiting the narrow coast strip of southeastern Alaska. No dispute has ever arisen as to that part of the boundary defined as being the 141st meridian of longitude west of Greenwich. As is well known, however, a contention arose as to that part of the boundary which delimits the stretch of coast extending from the neighborhood of Mt. St. Elias southeasterly to and through the Portland Canal. A modus vivendi in 1878, affecting the Stikine River, and another in 1899, relating to the country at the head of Lynn Canal, made temporary provision for customs and police purposes. The dispute relating to that part of the boundary was happily settled by the Tribunal of London which was constituted under a convention signed at Washington January 24, 1903.

Hon. John W. Foster, the agent of the United States in this important case, remarks in his report to Secretary Hay:

> "It is a noteworthy fact that this important adjudication was brought to a close within less than eight months from the time when the treaty creating the tribunal went into effect. Such a prompt result is almost without parallel in the intercourse of nations."

Equally prompt was the action of the governments in appointing commissioners in accordance with a requirement of the convention constituting the Tribunal. Within a few months, that is, in the spring of 1904, the commissioners, Mr. W. F. King, on behalf of the British Government, and your speaker, representing the United States, began the delimitation of that part of the boundary which had been in dispute. The commissioners were guided in their plans by maps, accompanying the decision, on which the Tribunal had marked certain mountain peaks as being the mountains contemplated by the Treaty of 1825.

It is the business of the commissioners to identify the peaks, to establish their geographical position, to mark by visible monuments, wherever possible, the turning points in the line and such other points as may be necessary, and to describe and define the line between the points selected by the Tribunal. There was a stretch of about one hundred and twenty miles where the topographic information was insufficient, and there the commissioners were directed to make additional surveys and to select mountain peaks within certain prescribed limits to define the boundary. The commissioners decided to mark at once certain river crossings and the mountain passes and to connect all the boundary peaks by a continuous triangulation based on the trigonometric datum adopted by the Coast and Geodetic Survey for southeastern Alaska.

The boundary line, starting from the neighborhood of Mt. St. Elias, crosses that summit and other high peaks of the St. Elias Alps and the Fairweather Range. In general, it lies amid perpetual snow and ice except when it drops abruptly into the river valleys only to rise again into regions of perpetual snow. Finally, it reaches the head of Portland Canal and becomes a water boundary.

In the four years since work was begun on this portion of the boundary the commissioners have fixed trigonometrically all the peaks

except two near Mt. St. Elias and those in the region between the Whiting River and Devil's Thumb, and some of the peaks south of the Unuk River. The passes, valleys and river crossings have been monumented with the exception of the crossing of the Alsek in the north and the valleys of the affluents of the Iskut and the crossing of the Le Duc and Chicamin rivers in the south. The turning points of the water boundary in Portland Canal also remain to be fixed by reference to points on shore.

The 141st Meridian.

According to the Treaty of 1825 the 141st meridian west of Greenwich forms the eastern boundary of Alaska from the Arctic Ocean to near Mt. St. Elias. It was not until 1889—twenty-two years after the acquisition of Alaska—that any steps were taken by our government towards establishing the location of the 141st meridian on the ground. In that year the Coast and Geodetic Survey despatched one party to the Yukon and another to the Porcupine River to determine the boundary crossing of those rivers. The Canadian government had previously sent an engineer to the Yukon who made an astronomical determination of the boundary in the autumn and winter of 1887. The country at that time was very inaccessible and the surveyors were compelled to determine the longitude by moon culminations and occultations, and the American parties spent a whole winter in observing them. But the operations of the three parties were not carried on under an international agreement and the results therefore were not reciprocally binding on the governments concerned.

The discovery of gold and the general development of the country, however, caused the construction of a Canadian telegraph line overland to Dawson and beyond, and later the United States government laid a cable from Seattle to Sitka and thence to Valdez on Prince William Sound, whence an overland line was built by the United States War Department as far as Fort Egbert on the Yukon near the boundary. Egbert and Dawson were also connected by telegraph. This important auxiliary to longitude determination made it possible for the two governments to determine the position of the 141st meridian with all the necessary accuracy as soon as an

agreement was reached and embodied in the treaty signed at Washington in August, 1906. This treaty provided for the survey and demarcation of the line and before the end of that summer the telegraphic determination of the Yukon River crossing of the 141st meridian had been completed. The telegraphic determination made by the Americans rests on the known longitude of Seattle. Signals were exchanged between Sitka and Seattle, Seattle and Valdez, Valdez and Fort Egbert, Fort Egbert and the boundary. That made by the Canadians rests on the longitude of Vancouver between which place and the boundary time signals were exchanged. As the difference between Seattle and Vancouver was also determined by the commissioners, the circuit was closed and a very satisfactory agreement was obtained.

It is worthy of remark in passing that the tracing of a meridian or parallel on the ground involves considerations which do not become apparent by an inspection of an artificial globe on which these lines are traced as smooth and regular curves. A parallel of latitude must be determined by astronomical observations, but in general the circumference of a small circle of the earth parallel to the equator will not lie in the same astronomical latitude, owing to the so-called deflection of the vertical. A series of points determined astronomically as being in the same latitude or, as in the case of a meridian, in the same longitude, will therefore in general produce on the surface of the earth a zig-zag line when they have been joined together.

In order to avoid all questions that might arise from local deflections of the zenith, it was provided by the Treaty of 1906 that the commissioners should determine by the telegraphic method a convenient point on the 141st meridian and then trace a north and south line passing through the point thus ascertained. This provision fixed the telegraph crossing of the boundary as the initial point for the longitude determination. The commissioners desired to make the determination as nearly on the 141st meridian as possible, in order to avoid a deflection error which might have been involved if the longitude had been obtained by linear measurement from a longitude observed at some distance from the boundary. The transit pier erected for the purpose of exchanging time signals was found

to be in longitude 141° 00′ 00″.4, a very close hit. It is interesting to note also that the final longitude differed only 9″.43 of an arc, or 410 feet, from that derived by moon culminations about twenty years before.

The work of tracing the boundary southward from the Yukon was begun in the spring of 1907 and was carried southward a distance of about one hundred and twenty miles. Aluminum-bronze monuments were erected on the north and south banks of the river, a trigonometric and topographic survey was made extending two miles on each side of the boundary for a distance of about forty-five miles, and a broad vista was cut through the woods for the same distance. The work planned for the coming season will carry the tracing of the line as far as the great mountains south of the White River, and the topographic survey and the monumenting will be pushed until the severity of the weather compels the surveyors to abandon the work and turn their faces homeward.

THE MOST PRIMITIVE LIVING REPRESENTATIVE OF THE ANCESTORS OF THE PLANT KINGDOM.

By GEORGE T. MOORE, Ph.D.

(*Read April 25, 1908.*)

There is but little doubt among botanists that the land flora as it now exists has originated from aquatic ancestors. Both from the morphologic and palæontologic standpoints the evidence corroborates this view. Indeed, the dependence of land plants upon an adequate water supply, together with the fact that in such groups as the Mosses and Ferns, fertilization itself can only be accomplished in the presence of water supplied from some external source, gave rise to the conclusion that the origin of the vegetable kingdom was from primitive plants living in the water, long before there was the more conclusive evidence now existing.

It would be interesting to inquire into the life histories of certain transitional groups with a view to tracing this migration from water to land. For modern morphological and physiological investigations has enabled us to do this with a considerable degree of certainty. Not only would we be able to show that the establishment of the higher representatives of our land flora had been brought about by certain methods of specialization in lower aquatic or semi-aquatic forms, but it would be possible to indicate to a certain extent at least how this process had been carried on. However such an inquiry would lead us entirely too far afield at this time and it will be necessary to grant without further discussion that the facts are sufficient to sustain the aquatic origin of the higher plants.

Naturally, in seeking for the primitive ancestors of the vegetable kingdom, attention is at once directed to the algæ, the group of plants which to a very considerable extent is more dependant upon the presence of external water for the carrying on of its vital processes than any other. Furthermore, in the present state of our knowledge, such an investigation would not be devoted to the more

highly differentiated brown or red algæ, but rather to the green algæ, in which group there exists the closest resemblances to the structure of the lower land plants. The problem thus becomes one of discovering as nearly as may be possible the most primitive member of the green algæ. And by "*primitive,*" of course, is not necessarily meant the simplest form, but that plant which seems to be nearest to the starting point of the phylogenetic tree and from which certain definite lines of ascent can be traced.

In considering the origin of the green algæ, numerous theories have been held and it would be impossible to give even a mere outline of the various improbable suggestions which have been advanced regarding the evolution of this group. During the past ten years, however, a great deal of light has been thrown upon the phylogenetic relationship of the algæ. Not only has the increase in our knowledge of the life histories of the algæ been considerable, but the discovery of many new genera and species has made clear the affinities of various families as never before. Of the 275 good genera now recognized among the green algæ, one fourth have been discovered and described since the appearance of Engler and Prantl's "Pflanzenfamilien"—the last complete work on the subject and still the recognized authority. The addition of so many new and in many cases important links to the chain of development of these plants, has reduced the former chaotic condition to something like order and it is no longer quite such a matter of speculation regarding the origin of the main group of the green algæ.

Ten years ago Chodat derived the green algæ from the simplest, unicellular, non-motile forms then known, namely, the Palmellaceæ. Within this family he included four genera whose simple life history showed three principal stages. From these so-called "conditions," as Chodat pointed out, developed the three important and ruling tendencies which have dominated the lower green algæ.

These are: (1) The *zoöspore condition,* or the unicellular motile stage, with the other two conditions transient or subordinate. (2) The *sporangium condition,* that is, the unicellular *non*-motile stage, with the other two conditions accidental or transient. (3) The *tetraspora condition,* where the non-motile cells are connected at right angles by the increasing consistency of the walls, giving rise

to the formation of a tissue or filament. The other two conditions are reduced or transient.

Having established these three principal "conditions," Chodat proceeded to establish the phylogeny of the green algæ along these lines and succeeded in clearing up considerable obscurity which had previously existed. However, the starting point selected by Chodat has been open to some criticism and it remained for Blackman to suggest the most satisfactory explanation of the origin of this group. He, while following in a general way the theory of Chodat, took the position that the three "tendencies" had their origin *not* in the non-moile *Palmella* form, but in the motile *Chlamydomonas* type.

I have had the genus *Chlamydomonas* under investigation for several years, observing its various species for the most part in pure cultures grown upon both solid and liquid media. The vegetative cells of *Chlamydomonas* are variable in both size and shape; in general, however, they are from 20–35 μ in length and 10–20 μ in breadth, being elliptic or pyriform in outline. One end of the cell is usually produced into a colorless beak, from which two cilia always protrude. The chloroplast is quite variable in form and with one exception is provided with a single pyrenoid. Non-sexual reproduction is by means of zoöspores, which are formed by the division of the contents of the mother cell, after it has come to rest. Sexual reproduction is usually by the conjugation of naked motile gametes of similar size and in no way distinguishable from each other. It is interesting to note, however, that in addition to this method there may also be the conjugation of unequal motile gametes and in one species—to be referred to later—there takes place the conjugation of dissimilar gametes, one of which, the larger, comes to rest before conjugation. We thus have within the limits of this well defined and natural genus, not only the most primitive form of gamogenesis, but through anisogamous conjugation a gradual approach to true oögamy—the highest type of sexual reproduction developed among the algæ.

In abandoning the starting point of Chodat's theory of the development of the green algæ, it is not necessary to replace his idea relative to the three predominating tendencies manifest in the lower members of this group. While different names are attached to these

conditions as recognized at the present time, they are essentially those pointed out by Chodat, namely:

1. A tendency towards the aggregation of motile vegetative cells, with a gradually larger and more specialized motile colony. This is the *Volvox* type and in no place in the plant kingdom do we have a more perfect series of development than from the simple *Chlamydomonas* form to the complex and highly differentiated *Volvox* type.

2. A tendency towards the formation of an aggregation of non-motile cells into a filament or tissue by the repeated vegetative division of an original mother cell. This is the *Tetraspora type.*

3. The *Endosphæra* type, where the tendency towards the formation of vegetative divisions and septate cell formation is reduced to a minimum. This is, of course, Chodat's *sporangium* tendency, although not so much importance is attached to it.

Without going into details it may be said that various species of *Chlamydomonas* (of which there are about thirty, all remarkably constant as regards their cytological characters), taken collectively, exhibit all these three tendencies and that the simpler forms of algæ which possess but a *single* tendency, seem clearly to have diverged from some one species of this genus.

The endosphærine tendency in *Chlamydomonas* has given rise to a single family, *Endosphæra.* This is naturally strictly unicellular and with no vegetative divisions; the reproduction of the species can take place only by the formation of zoöspores or gametes. A family so restricted as to its vegetative habit could hardly be expected to develop very far and it is interesting to note that practically all the genera are epiphytic upon other algæ or aquatic plants, and that this habit of life has undoubtedly given rise to a distinct group of fungi. The suggestion has been made that the peculiar Siphonales may have developed from this *Endosphæra type,* and while such a view is reasonable, it must necessarily, at the present time, be a mere matter of speculation.

But one family, the Volvocaceæ, has resulted from the development of the volvocine tendency. While the evolution of sex in this group has been carried to the highest possible degree, the restrictions of an enforced motile vegetative condition did not permit this family to give rise to anything further.

It is the tetrasporine tendency which has been the permanent one and has resulted in producing the higher green plants. This condition in *Chlamydomonas* resulted in the production of a series of plants which gradually replaced the formation of zoöspores by that of vegetative cell division. The resulting family was the Palmellaceæ, the one which formed the starting point in the development of the algæ, according to Chodat. While there seems to be but little question that the Palmellaceæ have given rise to most of the other families of the green algæ, there is every evidence that it was itself derived from *Chlamydomonas,* rather than the reverse, as contended by Chodat.

It is impossible at this time to even indicate the development of the higher algæ from the Palmellaceæ. With the exception of the Confervales, which seems to have developed independently of the typical green algæ, and the Conjugales, which apparently have arisen directly from the *Chlamydomonas* type, all the higher green algæ can be traced back through the Palmellaceæ with considerable certainty to their *Chalmydomonas* ancestor. The Conjugales have always been a stumbling block in constructing any developmental line of the algæ from primitive forms. But granting that the filamentous Conjugales, as well as the desmids, are unicellular (the reasons for which can not now be given) it is comparatively easy to find the origin of the conjugation habit so emphasized in this group, in *Chlamydomonas Braunii.* In this species the female or receptive cell comes absolutely to rest before fertilization and the smaller or male cell becomes attached to it. Then the entire contents of the male cell passes into the female cell, leaving behind the empty cell wall, just as in some of the desmids and in most of the filamentous-like Conjugales.

The more the genus is studied the more reasonable becomes the conclusion that *Chlamydomonas* has not only given rise to such an aberrent group as the Conjugales, but that it may safely be regarded as the phylogenetic starting point of the various lines of ascent in the true green algæ. At present in the higher algæ the *Chlamydomonas* stage is, of course, retained in the zoöspore and the gamete, a vegetative non-motile generation being interpolated between either

a sexual or non-sexual motile stage. In the very highest type of reproduction in the green algæ the male gamete alone represents the *Chlamydomonas* stage. Indeed, we may well assume that the motile male gamete of the mosses and ferns constitutes the last remaining type of the original *Chlamydomonas* condition, which with other more positive evidence points to the origin of such land forms from a *Chlamydomonas*-like ancestor.

Although practically all the evidence for the position taken has necessarily been omitted, it is hoped that enough has been said to at least indicate the unique and important position occupied in the plant kingdom by the alga *Chlamydomonas.*

THE COMPARATIVE LEAF STRUCTURE OF THE SAND DUNE PLANTS OF BERMUDA.

(With 3 plates.)

By JOHN W. HARSHBERGER, Ph.D.

(*Read April 24, 1908.*)

The writer has discussed the flora of Bermuda in two papers published in the *Proceedings of the Academy of Natural Sciences of Philadelphia* and entitled "The Plant Formations of the Bermuda Islands" (1905: 695–700) and "The Hour-glass Stems of the Bermuda Palmetto" (1905: 701–704). The study of the flora presented in these papers and the study of the microscopic anatomy of the leaves of the sand dune plants herewith given is the result of a visit to the islands during the month of June, 1905.

The sand beaches and sand dunes are found typically developed along the south shore of the main island and in a few isolated places on the north shore, as at Shelly Bay. The largest sand beaches and sand dunes on the south shore are found in the vicinity of Tuckertown Bay, on the narrow strip of south shore between Harrington Sound and the ocean. The sand dunes along the south shore in the parish of Paget are also characteristic. The sand dunes, however, in the neighborhood of Tuckertown Bay are remarkable in that they have encroached on the rocky shore line and have invaded the natural arch which is one of the scenic wonders of the islands. The sand has drifted beneath the arch and has advanced so that it covers part of the top of the arch itself.

The vegetation of the beaches and dunes here and in the vicinage of the Devil's Hole is characteristically Bermudian, while the sand dunes in Paget have been colonized in part by plants introduced by man into the islands, such as the oleander, *Nerium oleander,* and a tall fennel, *Fœniculum vulgare.* These beaches and dunes are formed of coral sand which represents the finely ground masses of

coral and coralline (calcareous) sea-weeds which have grown on the fringing coral reefs. Bermuda, geologically speaking, is an atoll, a ring of coral reefs surrounding a central lagoon. The elevated land was formed by the raising of the weather edge of the reef above the level of the sea. The tops of the projecting corals were broken off and along with calcareous sea-weeds and mollusk shells were ground by surf action into a fine sand, which was formed into a beach. As the top of the beach dried in the sun, the sand was blown off and was deposited in the crevices of the coral breakwater, which gradually widened. Ultimately, by wind action, sand hills were formed. The limestone rock found throughout the islands was originally derived from broken-down coral and shells. These rocks vary in texture from loose sand to compact limestone. The process by which the coral sand was converted into limestone was very simple and it involved no great lapse of time. As the sand consists almost entirely of calcium carbonate, it was easily soluble in water containing carbon dioxide. The rain water took up a little of the calcium carbonate in the form of bicarbonate, and as it percolated through the sand, it lost its carbonic acid gas and evaporating left the dissolved calcium carbonate as a thin layer of cement uniting together the grains of sand. The rocks remain permeable to water and soluble, so that this process of solution and deposition goes on constantly until even a marble-like limestone may result. The usual building material consists of blocks of limestone sawed out of the hillside. When built as a wall sufficient solution takes place so that the stones become united together into an almost solid piece. The red soils of the islands represent the one per cent. residue of solid material after the rain has leached out all of the other constituents. When the solution, owing to wave action or constant rain action, is excessive, caverns with stalactites and sinks are formed. The honey-combed eolian rock of the shore line on which characteristic Bermuda plants occur owed its origin to similar water erosion. The sand dunes thus represent stages intermediate in the geologic changes which have combined to give the present form to the islands. They represent shifting masses of coral sand, forming flat surfaces in some places, in other places heaped into conical dunes or raised into long ridges. Frequently dune hollows exist as a result of wind

action in scooping out the sand. These dunes form the setting upon which the typical sand strand plants are distributed.

PLANT DISTRIBUTION.—The upper beach at the foot of the dunes is characterized by the presence of *Cakile æqualis,* which shows a more decided branching habit than the closely related species on the coasts of the American continent, *Cakile maritima.* Besides this plant, the botanist sees clumps of *Tournefortia gnaphalodes, Scævola Plumieri* and *Croton maritimus.* The shrubs, however, grow most luxuriantly on the slopes and summits of the dunes. *Ipomæa pes-capræ,* as elsewhere in the tropics (Mexico, the West Indies), is a typical plant of the upper beach; in fact, the upper beach is characterized by its presence, with its long runners growing down from the slopes of the dunes out upon the flat, sandy beaches. On the dune slopes in Bermuda it is associated with *Scævola Plumieri* and the crab grass, *Stenotaphrum americanum.*

Back of the dune crests are found *Tournefortia gnaphalodes, Ipomæa pes-capræ, Scævola Plumieri, Juniperus bermudiana* (wind-swept forms), *Sisyrinchium bermudianum, Lepidium virginicum, Euphorbia buxifolia* (a prostrate plant growing in rosettes), *Canavalia obtusifolia* (a leguminous vine) and the prickly pear cactus, *Opuntia vulgaris.* On the dunes at Tuckertown, where the sand covers the entrance to the natural arch, *Scævola Plumieri* forms extensive clumps in pure association. *Solidago sempervirens,* as in the eastern United States, is also a dune plant, together with the smooth and hairy forms of *Borrichia arborescens, Dodonæa viscosa,* a small tree with its varnished leaves, is also a tenant of the dunes. The most interesting dune plant is *Conocarpus erectus,* which is a typical mangrove tree growing with its roots affected by salt water. In Bermuda, however, it occurs perhaps more frequently on the dry upper slopes of the dunes. In one place on the south shore, it covers nearly a quarter of an acre. The crab grass, *Stenotaphrum americanum* forms close mats on the lee side of the dunes.

The high dunes on the south shore of the parish of Paget have been invaded by a number of exotic plants, introduced by man into the islands, such as *Nerium oleander, Lantana camara, L. crocea,* while *Croton maritimus, Canavalia obtusifolia, Dodonæa viscosa, Borrichia arborescens* and *Passiflora suberosa* are among the most

abundant native plants. *Yucca aloifolia* forms clumps on low sand dunes at Shelly Bay, on the north shore, associated with *Ipomæa pes-capræ, Tournefortia gnaphalodes* and *Opuntia* sp.

Ecologic Factors.—The ecologic factors, which have influenced the distribution of the typical sand strand plants of Bermuda, must be referred to briefly. As the plants of the Bermuda sand beaches and sand dunes in general show xerophytic adaptations, we must look upon these adaptive arrangements as a response to the environment. The following environmental factors must be considered as influential in producing the xerophytic structures which the leaves of the Bermuda beach and dune plants especially show:

1. The intense illumination from above is an important ecologic factor.

2. The reflection of light from the white coral sand and the foam-crested breakers beyond is important.

3. The action of the strong winds that blow across the islands must be considered as modifying plant structure.

4. The action of the salt spray blown inland by the wind is marked in the case of some plants.

5. The permeability of the sand to water, so that after a rain the surface layers quickly dry out, has its influence.

The most potent factor in the modification of leaf structure has been undoubtedly the bright illumination from above and below (by reflection) and the physiologically dry condition of the soil.

Structural Adaptations.—The leaf adaptations to light are found in the increased number of palisade layers, their presence on the upper and under sides of the leaves, and their arrangement, so that the central part of the leaf becomes palisade tissue throughout, a typical staurophyll. The depression of the stomata below the surface, as in *Sisyrinchium bermudianum,* the distribution of the stomata in pits, as in *Nerium oleander* and *Lantana involucrata,* the development of hairs as in *Tournefortia gnaphalodes,* the varnished leaves of *Dodonæa viscosa* and thick epidermal layers and cuticle are all arrangements to reduce transpiration. The succulency of the leaves of some of the dune plants is developed perhaps for water storage and the presence of latex should be mentioned as a means by which

a dune plant is protected against the untoward influences of its environment.

Light has been most marked in influencing the development of leaf structure displayed by the typical sand dune plants of Bermuda. The stimuli of light have called forth functional responses which have produced changes in form or structure of the leaves, or in both. The chlorenchyma, composed of chloroplast-bearing cells, is converted into two kinds of tissues, palisade and spongy parenchyma, as a direct result of the unequal illumination of the leaf surfaces. Palisade tissue is formed as a response to light, or to low water content, or to both. When both leaf surfaces are equally illuminated, the leaf may be termed isophotic, when unequally illuminated diphotic. Diphotic leaves which are unequally illuminated show a division into palisade and spongy parenchyma, and such leaves are called by Clements[1] diphotophylls. Isophotic leaves, equally illuminated on both surfaces have a more or less uniform chlorenchyma. Clements divides such leaves into three types: (1) The palisade leaf, or staurophyll in which the palisade tissue extends from the lower to the upper epidermis. (2) The diplophyll, or double leaf, where the intense light does not penetrate to the middle of the leaf. In consequence, the upper and lower palisade layers are separated by a central loose parenchyma, which is for water storage. (3) The spongophyll, in which the rounded, loose parenchyma cells fill the leaf without palisade tissue. The influence of the light and other environmental conditions on leaf structure is perhaps best shown in the thin and thick leaves of *Conocarpus erectus* produced on different parts of the same tree differently related to the incident rays of light. A detailed description of these structures for each plant will be given at the end of the paper. The following is a classification of different leaf structures and the plants which illustrate such adaptive arrangements:

Thick Cuticle.—*Nerium oleander, Conocarpus erectus* (thin leaf), *Scævola Plumieri.*

Thick Epidermis.—*Canavalia obtusifolia, Dodonæa viscosa, Sisyrinchium bermudianum, Stenotaphrum americanum, Ipomæa pes-*

[1] Clements, F. E., "Research Methods in Ecology," 138–145; "Plant Physiology and Ecology," 171–184.

capræ, Cakile æqualis, Borrichia arborescens (smooth leaf), *Croton maritimus.*

Two or Three Epidermal Layers.—*Euphorbia buxifolia, Nerium oleander, Conocarpus erectus* (thick leaf), *Croton maritimus, Tournefortia gnaphalodes.*

Two or More Rows of Palisade Cells.—*Passiflora suberosa, Dodonæa viscosa, Nerium oleander, Sesuvium portulacastrum, Cakile æqualis, Conocarpus erectus* (thin leaf and thick leaf), *Scævola Plumieri, Borrichia arborescens* (smooth and hairy leaves).

Stomata Depressed.—*Sisyrinchium bermudianum Heliotropium curassavicum, Sesuvium portulacastrum, Ipomæa pes-capræ, Cakile æqualis, Conocarpus erectus* (thick leaf), *Scævola Plumieri, Borrichia arborescens* (smooth leaf).

Stomata in Pits.—*Lantana involucrata, Nerium oleander.*

Succulent Leaf.—*Sesuvium portulacastrum, Cakile æqualis, Conocarpus erectus* (thick leaf), *Scævola Plumieri, Borrichia arborescens* (smooth leaf).

Hairy Leaf.—*Lantana involucrata, Nerium oleander, Borrichia arborescens* (hairy leaf), *Croton maritimus, Tournefortia gnaphalodes.*

Varnished Leaf.—*Dodonæa viscosa.*

Leaf Becoming Erect in Sun Position.—*Canavalia obtusifolia, Sisyrinchium bermudianum, Stenotaphrum americanum, Ipomæa pes-capræ.*

Overlapping Leaves.—*Euphorbia buxifolia, Sisyrinchium bermudianum, Stenotaphrum americanum.*

Latex Tissue.—*Euphorbia buxifolia.*

Gum-Resin.—*Conocarpus erectus.*

Crystals.—*Passiflora suberosa, Croton maritimus.*

Diphotophyll.—*Passiflora suberosa, Canavalia obtusifolia, Euphorbia buxifolia, Lantana involucrata, Nerium oleander* = 5.

Diplophyll.—*Dodonæa viscosa, Sesuvium portulacastrum, Ipomæa pes-capræ, Cakile æqualis, Conocarpus erectus* (thin leaf), *Scævola Plumieri,*[2] *Borrichia arborescens* (smooth and hairy leaves), *Croton maritimus, Tournefortia gnaphalodes* = 9.

[2] *Scævola Plumieri* and *Tournefortia gnaphalodes* are given twice, because it is difficult to decide whether their leaves are diplophyll, or staurophyll.

Staurophyll.—*Heliotropium curassavicum, Conocarpus erectus* (thick leaf), *Scævola Plumieri, Tournefortia gnaphalodes*[2] = 4.

Spongophyll.—*Sisyrinchium bermudianum, Stenotaphrum americanum* = 2. With reference to the last two plants, it should be mentioned that the leaves of these plants stand erect, thus receiving the incident rays of light on the edge of the leaf, hence the absence of palisade tissue and the presence of spongophyll structure.

Detailed Structure of Leaves.—The sections of the leaves which were studied were made free-hand with a razor. After staining, the sections were mounted for permanency in Canada balsam. The drawings of these sections were made by the use of the microprojection, electric lantern, so that in every case with the exception of *Croton maritimus,* the drawings were made on the same scale. The sketches of stomata are none of them drawn to the same scale. The description of the histologic structure of the leaves of each species follows.

Passiflora suberosa is a small, slender species of the genus found growing over the sand surface of the dunes in the parish of Paget. Its flowers are small and the branch tendrils are characteristically developed. Histologically the leaf presents an upper epidermis of large thin-walled cells, and as the whole plant is brilliantly illuminated, it has two well-marked layers of palisade cells. The loose parenchyma is narrow and some of the cells of it are filled with conglomerate crystals. The stomata are slightly raised above the general epidermal surface, and are confined to the lower side of the leaf. A diphotophyll (Fig. 1, Plate II.).

Canavalia obtusifolia, a trailing leguminous plant, has paripinnate compound leaves with a long petiole and broadly elliptical leaflets with retuse apices and petiolules, a quarter of an inch long. The upper epidermis consists of slightly thickened cells. There are two rows of palisade cells, a considerable amount of loose parenchyma, while the slightly raised stomata are found on the upper and under sides. The adaptation to the environment of the sand dunes seems to be the folding together of the two sides of the leaves along the midrib, so that the edges of the leaves are presented to the incident rays of light. A diphotophyll (Fig. 2, Plate II.).

Euphorbia buxifolia is a prostrate, tufted plant of a rosette habit.

The taproot is large and strong and from it numerous branches, six to eight inches long, are formed. The leaves are opposite, small, ovate, with an acute apex and barely petiolate. The upper epidermis consists of two rows of cells, the palisade is a single layer and the loose parenchyma is compact. The lower epidermal cells are papillate and latex is present. The adaptation to the environment is shown in the latex, the two-layered upper epidermis and the overlapping arrangement of the leaves. A diphotophyll (Fig. 3, Plate II.).

Dodonæa viscosa.—This small sapindaceous tree occurs on the inner edges of the sand dunes. Its leaves are alternate, spatulate with the base narrowed to the point of attachment. The leaves are varnished. The upper epidermal cells are thick and provided with peltate hairs. The palisade cells are disposed in two layers. The loose parenchyma is open, while next to the lower epidermis there is a row of small cells which may be considered as a lower palisade layer. Hence the leaf is a potential diplophyll. The stomata of the upper side are slightly raised above the surface, while those on the under side have developed a small projecting beak (Fig. 4, Plate II.).

Lantana involucrata is one of the plants that enters the formation of the Bermuda scrub. It also invades the dunes. The leaves are hairy on both surfaces. A section of a leaf shows that the upper epidermis is without stomata, but is provided with straight, multicellular and capitate, unicellular hairs. The lower surface shows depressions provided with the capitate hairs, while the raised portions of the leaf surface between the depressions is covered with both straight, muticellular and capitate, unicellular hairs. The palisade is a single layer. The stomata project outward beyond the general surface of the lower epidermis, but they always occur in the depressions. The depressions provided with hairs and stomata and thick, hairy upper epidermal surface are structures which fit the plant to exist on the hot, sun-exposed sand dunes of the islands. A diphotophyll (Fig. 5, Plate II.).

Nerium oleander.—The leaf structure of the oleander, a native of the Mediterranean flora, is well known. The upper epidermis is in three layers with thick cuticle, the paliside tissue in two layers, while the under surface of the leaf is pitted, the pits being filled

with straight hairs that form an air-still chamber into which the projecting stomata open. The lower epidermis is two- to three-layered, and the whole leaf is decidedly tough and leathery, and thus well adapted to growing on the sand dunes of Bermuda. A diphotophyll (Fig. 6, Plate II.).

Sisyrinchium bermudianum.—The Bermuda blue-eyed grass is provided with leaves that stand more or less upright, so that the incident rays of light strike the edges of the leaves. The epidermal cells on both the upper and lower morphologic sides of the leaf are thick-walled and the stomata present on both surfaces are depressed the entire width of the epidermal cells. There is no palisade tissue, the loose parenchyma filling the center of the leaf between epidermal surfaces. The vertical leaves are, therefore, isophotic and the leaf is known as a spongophyll. The vertical leaves, the thick epidermal cells and the depressed stomata fit the plant to its environment. A spongophyll (Fig. 7, Plate II.).

Stenotaphrum americanum. The Bermuda crab grass is a tough, wiry one, well fitted to survive in the driest places on sand dunes and rock faces. The leaf blades arise from sheaths that, together with other overlapping leaf sheaths, form a tuft that arises from the nodal regions of the wiry, prostrate, creeping stem. The blades are more or less erect and folded partially lengthwise, with the upper side innermost. The spike of closely set flowers is slightly bent, suggesting a crab's claw. The upper epidermis consists of large, open papillate cells. The loose parenchyma fills the leaf section and the under surface of the leaf has a thick epidermis with numerous stomata, provided with small guard cells reënforced by two secondary cells. The bundles are toward the upper side. The vertical isophotic leaf consequently becomes a spongophyll. The adaptations to the environment are upright, rolled leaves, thick lower epidermis and overlapping, tufted leaf sheaths (Fig. 8, Plate II.).

Heliotropium curassavicum resembles in its unilateral cymose inflorescence the common heliotrope. It is a slightly woody plant that grows about a foot or two tall, with alternate, narrow, oblanceolate leaves. The cells of both the lower and upper epidermis are thin-walled, with slightly sunken stomata on both sides. The chlorophyll bearing cells of the leaf (the chlorenchyma) are arranged so

that their long axes are placed in a line with the incident rays of light that strike the upper surface from above and the lower surface by reflection from the sand below. A staurophyll (Fig. 9, Plate II.).

Sesuvium portulacastrum.—The leaf structure of this member of the family Aizoaceæ is that of a typical diplophyll, but with a slight indication of the staurophyll arrangement of the cells. The stomata present on both sides of the leaf are slightly sunken and the guard cells incline inward and downwards. The upper and lower palisade tissues show four to five layers of cells. The leaves are thick and succulent. A diplophyll (Fig. 10, Plate II.).

Ipomæa pes-capræ.—This tropical, seaside morning glory is a typical plant of the sandy beaches in Mexico, the West Indies and Bermuda. It grows down off the dune slopes onto the beach sand as a creeping plant, a distance of twenty to thirty feet (Fig. 1, Plate I.). The leaves are alternate, elliptical, retuse at the apex and frequently when the sun is hot and the reflection from the sand intense, the leaves fold together along the midrib and stand vertically so as to receive the incident rays of light on the upturned edges of the leaves. The walls of the epidermal cells on both sides of the leaf are thick. The stomata on both sides are sunken about half the thickness of the epidermal cells and the palisade tissue is prominent on both sides, constricting the loose parenchyma to a narrow layer. The leaf is, therefore, a true diplophyll (Fig. 11, Plate III.).

Cakile æqualis.—This cruciferous plant grows on open, sandy beaches in a more or less scattered manner. It branches in a much more open way than *C. maritima,* found in similar habitats on the sandy beaches of the eastern United States. The leaves are fleshy and the walls of the upper and lower epidermal cells are thickened. The stomata, which are partly sunken, are found on both the upper and the lower leaf surfaces. The palisade tissue on both sides is five layers of cells thick and the loose parenchyma is restricted to a narrow layer four cells thick in the central part of the leaf. This plant is fitted to its environment by the possession of succulent leaves, epidermal cells with thick walls, and many-layered palisade tissue. A diplophyll (Fig. 12, Plate III.). Contrast the leaf section of *Cakile maritima* (Fig. 12 *A*, Plate III.).

Conocarpus erectus.—The leaves of this small tree, which is a true mangrove plant, but which has adapted itself to growth on the sand dunes in Bermuda, are thin and thick. The thin leaves are found on the branches that are placed above the surface of the sand, or in more or less protected positions, while the thick, succulent leaves occur near the surface of the sand, or in exposed, unshaded positions. There is a considerable difference in the anatomical structure. The cuticle in the thin leaf is thickened and the stomata on both sides are hardly if any sunken below the surface. The upper leaf surface shows long palisade cells, while the palisade cells of the lower side are shorter. The loose parenchyma cells form a broad band in the center of the section. A diplophyll (Fig. 13, Plate III.). The thick, succulent leaf has three rows of epidermal cells and three rows of palisade cells, the cavities of which are filled with a gummy, resinous material (not tested) of a brown color. This gummy material is found in the lower palisade as well as in the upper palisade in both the thin and thick leaves and also in some of the loose parenchyma cells of the thick leaf. The stomata in the thick leaves, by the increase in the thickness of the cuticle, are sunken below the surface with an hour-glass atrium or passage outside of the thick-walled guard cells. The parenchyma cells of the leaf center are arranged in the direction of the palisade cells. A typical staurophyll (Fig. 13 *A*, Plate III.).

Scævola Plumieri.—This plant belongs to the family Goodeniaceæ and forms dense clumps on the dune slopes (Fig. 2, Plate I.). Its leaves are alternate, elliptical, short petiolate and obtuse. They are noted for their succulency. The epidermal cells on the upper surface have a thick cuticle with numerous thick walled, sunken stomata. The epidermal cells on the lower surface are of the same thickness as on the upper surface, the stomata being likewise sunken. The palisade cells on the upper and lower sides consist in each of three or four rows of cells, while the loose parenchyma is arranged parallel to the palisade tissue. Only a single row of central cells are not so disposed. The leaf shows, therefore, partly a staurophyll and partly a diplophyll arrangement of cell (Fig. 14, Plate III.).

Borrichia arborescens.—This species of the family Compositæ exists in two distinct forms, if they are not good species. One form

has smooth, thick, succulent leaves, the other has thinner, densely tomentose leaves, the *Borrichia frutescens* of the Southern States. The succulent, smooth-leaved form has both thick upper and lower epidermal cells, with the stomata on both sides, but more plentiful on the lower side. The stomata are partly sunken. The palisade layers on both sides are wide, but are broken into more or less extended patches by round parenchyma cells, which reach to the epidermis. The loose parenchyma cells form a wide central area. A diplophyll (Fig. 15, Plate III.). What the thin leaf lacks in succulency, it gains in hairiness. Both sides are densely covered with straight unicellular hairs. The palisade layers are only two in number on both sides of the leaf, and the loose parenchyma is also much reduced in amount. The succulency of the thick leaf fits it as perfectly as the hairiness of the thin leaf to the trying seaside environment, where the plants producing them grow side by side. A diplophyll (Fig. 15 *A*, Plate III.).

Croton maritimus.—The leaves of this plant studied by Kearney[3] are bifacial, both surfaces densely covered with gray scale-like pubescence, owing to presence of multicellular, stalked, stellate hairs that cover them. The upper and lower epidermal cells have thick walls and the stomata are not sunken. The palisade tissue in both the upper and the lower sides are two cell layers in width with a few sclerotic idioblasts. The leaf in the plant grown in the United States, as depicted by Kearney, has only one row of palisade cells. Large conglomerate crystals of calcium oxalate are found in the cells of the loose parenchyma. Glandular capitate hairs are found on both leaf surfaces. A diplophyll (Fig. 16, Plate III.).

Tournefortia gnaphalodes.—The leaves and stems of this plant, as well as the calices of the flowers, are covered with a dense, closely appressed, grayish tomentum, resembling that on our common *Antennaria plantaginifolia* and edelweiss, *Leontopodium alpinum.* In section the hairs are unicellular, straight and of epidermal origin. The palisade is formed on the upper and lower leaf surfaces and is two cells thick. The loose parenchyma, occupying the center of the leaf, suggests an arrangement in direction parallel to the long axis

[3] Kearney, Thomas H. "Plants of Ocracoke Island," Contributions from the United States National Herbarium V: 296.

of the palisade cells. Therefore it is a diplophyll (Fig. 17, Plate III.).

BIBLIOGRAPHY.—Little has been published on the structure of the dune plants of tropical America. The following papers are in part a contribution to our knowledge of the microscopic structure of the strand plants of the American tropics. A few of the sand dune plants are of cosmopolitan distribution and they are, therefore, described as to their morphology in the classic work of A. F. W. Schimper, "Die indo-malayische Strandflora," published as the third volume of "Botanische Mittheilungen aus den Tropen" in 1891. Thomas Kearney in 1900 published in the Contributions from the U. S. National Herbarium (V., No. 5) an important paper on "The Plant Covering of Ocracoke Island; A Study in the Ecology of the North Carolina Strand Vegetation." A chapter is devoted to the histological structure of the plants. The only plants which concern us are *Yucca aloifolia, Croton maritimus, Borrichia frutescens,* which are common also to the Bermuda strand. F. Boergesen and Ové Paulsen make a contribution to "La Vegetation des Antilles Danoises" in Revue Générale de Botanique (Tome XII., 1900), in which they discuss with figures the microscopic structure of a few of the typical strand plants. As throwing considerable light on the problems concerned in this paper on the Bermuda strand flora reference should be made to these works of general import to the botanical questions involved.

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ILLUSTRATIONS.—The reproduced photographs (Figs. 1 and 2, Plate I.) represent the dune vegetation on the south shore of Bermuda. The upper illustration shows the thicket of composite vegetation on the crest of the dune and the long, trailing stems of *Ipomæa pes-carpæ* on the upper beach with a small clump of *Cakile æqualis* to the left in the foreground. The second illustration depicts a clump of *Scævola Plumieri,* with the Bermuda cedar, *Juniperus bermudiana,* and in the background the grayish-green bushes of *Tournefortia gnaphalodes.* Reference is made to the drawings of microscopic structure in the classified description of dune plants throughout the paper.

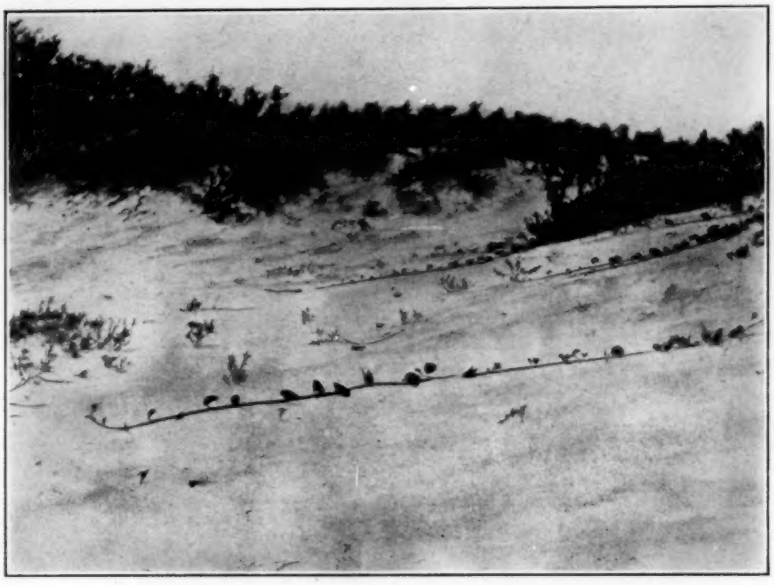

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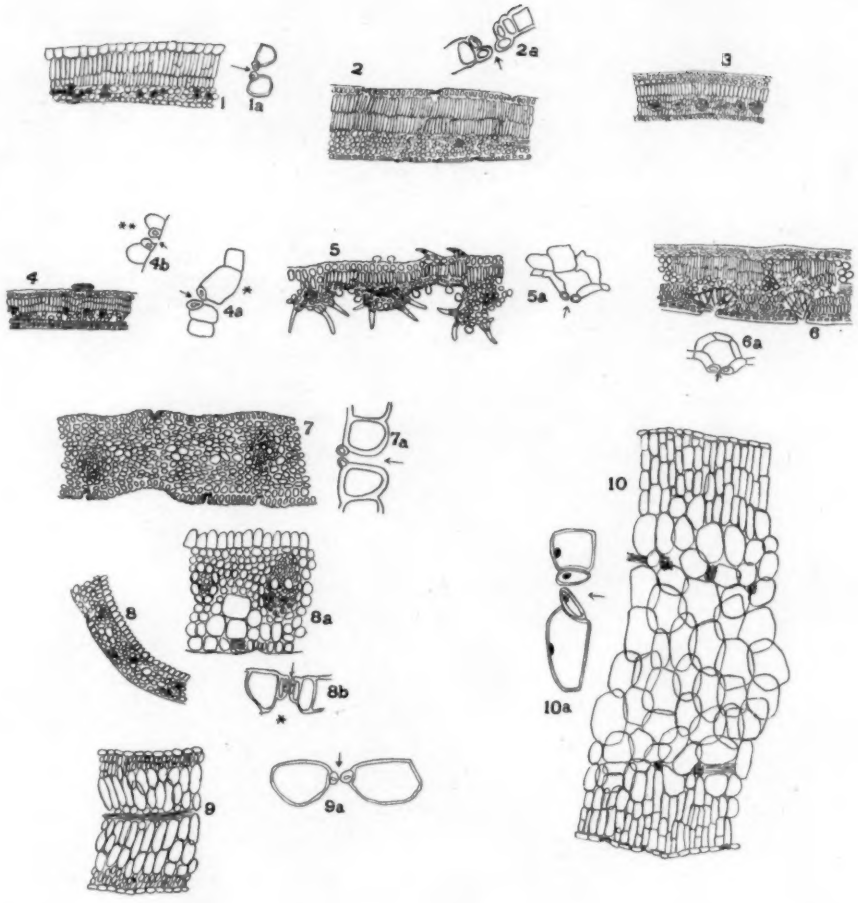
FIG. 2.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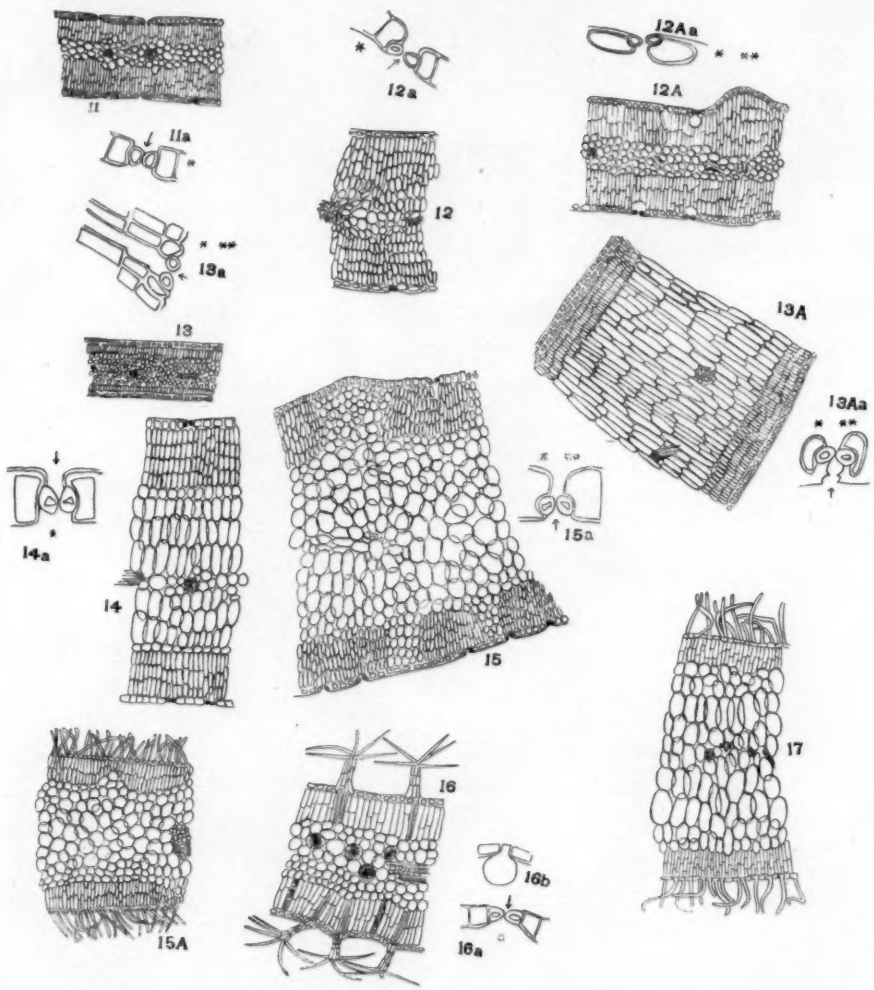
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SOLUTION OF ALGEBRAIC EQUATIONS IN INFINITE SERIES.

By PRESTON A. LAMBERT.

(*Read April 25, 1908.*)

I. Introduction.

1. The object of this investigation is to develop a method for determining all the roots, real and imaginary, of an algebraic equation by means of infinite series.

2. Suppose the given equation to be represented by $f(y)=0$. The method consists in introducing a factor x into all the terms but two of the given equation; expanding y, which now is an algebraic function of x, into a power series in x; placing x equal to unity in this power series. The resulting value of y, if convergent, is a root of the given equation expressed in terms of the coefficients and exponents of the equation.

3. The method presupposes the solution of the two-term equation

$$ay^n+b=0.$$

In fact the roots of this equation when written in the form

$$y^n=-\frac{b}{a}=r(\cos\theta+i\sin\theta)$$

are found to any required degree of approximation from the formula

$$y=r^{\frac{1}{n}}\left(\cos\frac{2s\pi+\theta}{n}+i\sin\frac{2s\pi+\theta}{n}\right),$$

where

$$s=0, 1, 2, 3, 4, \cdots, n-1.$$

4. The method proceeds step by step from the two-term equation to the three-term equation, from the three-term equation to the four-term equation, and so on.

II. The Three-term Equation.

5. In the three-term equation

$$ay^n + by^k + c = 0$$

the two terms from which the x is to be omitted can be selected in three different ways. This gives rise to the three equations

(1) $$ay^n + by^k x + c = 0$$

(2) $$ay^n + by^k + cx = 0$$

(3) $$ay^n x + by^k + c = 0$$

each one of which defines y as an algebraic function of x.

6. Values of y expressed as power series in x may be found from each one of these three equations by any one of the following three methods, which, however, are essentially the same.

7. *The Multinomial Theorem.*—Assume that the power series for y is

(4) $$y = p_0 + p_1 x + p_2 x^2 + p_3 x^3 + p_4 x^4 + \cdots.$$

The multinomial theorem asserts that the coefficient of x^r in the expansion of y^n is

(5) $$\Sigma \frac{n!}{q_0!\, q_1!\, q_2! \cdots q_s!} p_0^{q_0} p_1^{q_1} p_2^{q_2} \cdots p_s^{q_s} x^{q_1 + 2q_2 + 3q_3 + \cdots sq_s},$$

provided

(6) $$q_1 + 2q_2 + 3q_3 + \cdots sq_s = r$$

(7) $$q_0 + q_1 + q_2 + \cdots q_s = n.$$

The expansion of y^k is obtained in like manner.

Assuming that the power series (4) represents the algebraic function defined by equation (1), the substitution of the expansions of y^n and y^k in equation (1) must give an identity. This identity is

(8)
$$\begin{aligned} 0 = {} & ap_0^n + anp_0^{n-1}p_1 \\ & + c \quad + bp_0^k \end{aligned} \Bigg| x + \frac{an(n-1)}{1\cdot 2} p_0^{n-2} p_1^2 \Bigg| x^2 + \frac{an(n-1)(n-2)}{1\cdot 2\cdot 3} p_0^{n-3} p_1^3 \Bigg| x^3 + \cdots$$
$$\begin{aligned} & + anp_0^{n-1}p_2 \\ & + bkp_0^{k-1}p_1 \end{aligned} \Bigg| \begin{aligned} & + an(n-1)p_0^{n-2}p_1 p_2 \\ & + anp_0^{n-1}p_3 \\ & + \frac{bk(k-1)}{1\cdot 2} p_0^{k-2} p_1^2 \\ & + bkp_0^{k-1}p_2 \end{aligned} \Bigg|$$

In this identity the coefficient of each power of x equals zero. Hence p_0 is the root of the two-term equation

$$ap_0 + c = 0.$$

The coefficient of the first power of x equated to zero determines p_1 uniquely in terms of p_0; the coefficient of x^2 equated to zero determines p_2 uniquely in terms of p_0 and p_1; in general, the coefficient of x^s equated to zero determines p_s uniquely in terms of $p_0, p_1, p_2, \cdots, p_{s-1}$. All the successive coefficients of the power series (4) are therefore determined uniquely in terms of p_0, any one of the roots of the two-term equation $ap_0^n + c = 0$.

The power series representing the algebraic functions defined by equations (2) and (3) are determined in precisely the same manner. Unfortunately if the coefficients of the power series are determined in this way it is difficult to recognize the law which will enable one to write the general term of the power series, which is necessary for the application of a convergency test.

When x is made unity, the equations (1), (2) and (3) become the three-term equation

$$ay^n + by^k + c = 0$$

and the power series, if convergent when $x = 1$, becomes the solution of this equation.

If it is known in advance that some one of equations (1), (2), (3) furnishes a power series which is convergent when $x = 1$, the multinomial theorem determines in an elementary and direct manner the coefficients of the power series.

8. *Maclaurin's Series.*—The algebraic function y defined by the equation

$$ay^n + by^k x + c = 0$$

can be expanded into a power series in x by means of Maclaurin's series

$$(9) \qquad y = y_0 + \frac{dy_0}{dx_0}x + \frac{d^2y_0}{dx_0^2}\frac{x^2}{1\cdot 2} + \frac{d^3y_0}{dx_0^3}\frac{x^3}{1\cdot 2\cdot 3} + \cdots.$$

The expansion is identical in form with the expansion obtained by means of the multinomial theorem and consequently has the same disadvantage.

9. *Lagrange's Theorem.*—The equation

$$ay^n + by^k x + c = 0$$

may be written

$$y^n = -\frac{c}{a} - x\frac{b}{a}y^k. \tag{10}$$

Placing $y^n = z$, whence $y = z^{1/n}$, this equation becomes

$$z = -\frac{c}{a} - x\frac{b}{a}z^{\frac{k}{n}}. \tag{11}$$

Lagrange's theorem asserts that if

$$z = v + x\phi(z)$$

$$f(z) = f(v) + x\phi(v)f'(v) + \frac{x^2}{1\cdot 2}\frac{d}{dv}\{\overline{\phi(v)}^2 f'(v)\} + \cdots + \frac{x^n}{n!}\frac{d^{n-1}}{dv^{n-1}}\{\overline{\phi(v)}^n f'(v)\}\cdots. \tag{12}$$

If now

$$f(z) = z^{\frac{1}{n}}, \quad \phi(z) = -\frac{b}{a}z^{\frac{k}{n}},$$

and after the derivatives in series (12) have been formed v is replaced by $-c/a$, there results, making x unity,

$$z^{\frac{1}{n}} = y = \left(-\frac{c}{a}\right)^{\frac{1}{n}} + \frac{b}{nc}\left(-\frac{c}{a}\right)^{\frac{1+k}{n}} + \frac{b^2}{2!\,n^2c^2}(1+2k-n)\left(-\frac{c}{a}\right)^{\frac{1+2k}{n}} + \frac{b^3}{3!\,n^3c^3}(1+3k-n)(1+3k-2n)\left(-\frac{c}{a}\right)^{\frac{1+3k}{n}} + \frac{b^4}{4!\,n^4c^4}(1+4k-n)(1+4k-2n)(1+4k-3n)\left(-\frac{c}{a}\right)^{\frac{1+4k}{n}} + \cdots. \tag{13}$$

In series (13) the law of formation of the successive terms is evident and this law is readily proved by induction by using Lagrange's theorem.

Series (13) may be more concisely written by placing

$$\left(-\frac{c}{a}\right)^{\frac{1}{n}} = y_0,$$

so that y_0 is a root of the two-term equation

$$ay_0^n + c = 0,$$

and denoting the continued product

$$(1 + sk - n)(1 + sk - 2n)(1 + sk - 3n)\cdots[1 + sk - (s-1)n]$$

by

$$\left[\frac{1 + sk - n}{1 + sk - sn}\right]. \tag{14}$$

With these conventions series (13) becomes

$$\begin{aligned} y = y_0 &+ \frac{b}{nc} y_0^{1+k} + \frac{b^2}{2!\,n^2c^2}[1 + 2k - n] y_0^{1+2k} \\ &+ \frac{b^3}{3!\,n^3c^3}\left[\frac{1+3k-n}{1+3k-3n}\right] y_0^{1+3k} + \frac{b^4}{4!\,n^4c^4}\left[\frac{1+4k-n}{1+4k-4n}\right] y_0^{1+4k} + \cdots \\ &+ \frac{b^s}{s!\,n^sc^s}\left[\frac{1 + sk - n}{1 + sk - sn}\right] y_0^{1+sk} + \cdots. \end{aligned} \tag{15}$$

If series (15) is convergent, it will furnish a root of the three-term equation

$$ay^n + by^k + c = 0$$

for each one of the n values of y_0.

10. To test series (15) for convergency write the first n terms in regular order in a row, underneath this row the succeeding n terms and so on indefinitely. The terms of series (15) will now be arranged in n columns as follows:

(16)

$$y_0 + \frac{b}{nc}y_0^{1+k} + \cdots + \frac{b^{n-1}}{(n-1)!\,n^{n-1}c^{n-1}}\left[\frac{1+(n-1)k-n}{1+(n-1)k-(n-1)n}\right]y_0^{1+(n-1)k}$$

$$+ \frac{b^n}{n!\,n^n c^n}\left[\frac{1+nk-n}{1+nk-n^2}\right]y_0^{1+nk} + \frac{b^{n+1}}{(n+1)!\,n^{n+1}c^{n+1}}\left[\frac{1+(n+1)k-n}{1+(n+1)k-(n+1)n}\right]y_0^{1+(n+1)k} + \cdots + \frac{b^{2n-1}}{(2n-1)!\,n^{2n-1}c^{2n-1}}\left[\frac{1+(2n-1)k-n}{1+(2n-1)k-(2n-1)n}\right]y_0^{1+(2n-1)k}$$

$$+ \frac{b^{2n}}{(2n)!\,n^{2n}c^{2n}}\left[\frac{1+2nk-n}{1+2nk-2n^2}\right]y_0^{1+2nk} + \frac{b^{2n+1}}{(2n+1)!\,n^{2n+1}c^{2n+1}}\left[\frac{1+(2n+1)k-n}{1+(2n+1)k-(2n+1)n}\right]y_0^{1+(2n+1)k} + \cdots + \frac{b^{3n-1}}{(3n-1)!\,n^{3n-1}c^{3n-1}}\left[\frac{1+(3n-1)k-n}{1+(3n-1)k-(3n-1)n}\right]y_0^{1+(3n-1)k}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$+ \frac{b^{sn}}{(sn)!\,n^{sn}c^{sn}}\left[\frac{1+snk-n}{1+snk-sn^2}\right]y_0^{1+snk} + \frac{b^{sn+1}}{(sn+1)!\,n^{sn+1}c^{sn+1}}\left[\frac{1+(sn+1)k-n}{1+(sn+1)k-(sn+1)n}\right]y_0^{1+(sn+1)k} + \cdots + \frac{b^{sn-1}}{(sn-1)!\,n^{sn-1}c^{sn-1}}\left[\frac{1+(sn-1)k-n}{1+(sn-1)k-(sn-1)n}\right]y_0^{1+(sn-1)k}$$

$$\vdots \qquad \vdots \qquad \vdots$$

This rearrangement of the terms of series (15) into the n columns of the table is permissible, inasmuch as throughout this investigation only absolute convergence is considered.

Cauchy's ratio test shows that each one of the n partial series composed of the terms in each of the n columns of the table is convergent when

$$\frac{b^n}{a^k c^{n-k}} < \frac{n^n}{k^k (n-k)^{n-k}}. \tag{17}$$

11. In like manner, if the algebraic functions defined by the equations

$$ay^n + by^k + cx = 0 \tag{2}$$

$$ay^n x + by^k + c = 0 \tag{3}$$

are expanded into power series in x by Lagrange's theorem, and if x is made unity in this power series, it is found that the resulting infinite series are convergent, provided

$$\frac{b^n}{a^k c^{n-k}} > \frac{n^n}{k^k (n-k)^{n-k}}. \tag{18}$$

12. If condition (18) is satisfied, equation (2) determines $n-k$ and equation (3) determines k roots of the three-term equation

$$ay^n + by^k + c = 0.$$

Either condition (17) or condition (18) must be satisfied, unless

$$\frac{b^n}{a^k c^{n-k}} = \frac{n^n}{k^k (n-k)^{n-k}}. \tag{19}$$

If condition (19) is satisfied, Raabe's test shows that the series obtained from equations (1), (2), (3) are all convergent.

13. The convergency conditions for equations (1), (2), (3) may be written by following these directions:

(a) To the left of the sign of inequality stands a fraction whose numerator contains the coefficient of the middle term of the three-term equation

$$ay^n + by^k + c = 0$$

and whose denominator contains the product of the coefficients of the end terms, the exponent of each coefficient being the difference of the exponents in the other two terms taken in order from left to right.

(*b*) The fraction to the right of the sign of inequality is obtained from the fraction to the left by replacing each coefficient by its exponent.

(*c*) The sign of inequality is $<$ when the term containing x is between the other two terms; if the term containing x is an end term the sign of inequality is $>$.

14. The following table exhibits the convergency conditions for the series obtained from equations (1), (2), (3) and the number of roots of the three-term equation

$$ay^n + by^k + c = 0$$

furnished by each one of these series.

(20)

(1)	$ay^n + by^k x + c = 0$	n	$\dfrac{b^n}{a^k c^{n-k}} \leqq \dfrac{n^n}{k^k (n-k)^{n-k}}$,
(2)	$ay^n + by^k + cx = 0$	$n-k$	$\left\{ \dfrac{b^n}{a^k c^{n-k}} \geqq \dfrac{n^n}{k^k (n-k)^{n-k}}. \right.$
(3)	$ay^n x + by^k + c = 0$	k	

The roots of the three-term equation can always be expressed in infinite series.

III. The Four-term Equation.

15. In the four-term equation

$$ay^n + by^k + cy^l + d = 0$$

the two terms from which the factor x is to be omitted can be selected in six different ways. This gives rise to the six equations:

(21) $ay^n + by^k x + cy^l x + d = 0$

(22) $ay^n + by^k + cy^l x + dx = 0$

(23) $ay^n x + by^k + cy^l x + d = 0$

(24) $ay^n x + by^k + cy^l + dx = 0$

(25) $ay^n x + by^k x + cy^l + d = 0$

(26) $ay^n + by^k x + cy^l + dx = 0$

Each one of these six equations defines y as an algebraic function of x. The y of equation (21) may be expanded into a power series in x by any one of the three methods of articles 7, 8, 9. Using the symbol (14) and denoting $(-d/a)^{1/n}$ by y_0, this power series, when x is made unity, becomes

$$(27)\quad y = y_0 + \frac{b}{nd}y_0^{1+k} + \frac{c}{nd}y_0^{1+l}$$

$$+\frac{b^2}{2!\,n^2d^2}[1+2k-n]y_0^{1+2k}+\frac{2bc}{2!\,n^2d^2}[1+k+l-n]y_0^{1+k+l}+\frac{c^2}{2!\,n^2d^2}[1+2l-n]y_0^{1+2l}$$

$$+\frac{b^3}{3!\,n^3d^3}\left[\frac{1+3k-n}{1+3k-3n}\right]y_0^{1+3k}+\frac{3b^2c}{3!\,n^3d^3}\left[\frac{1+2k+l-n}{1+2k+l-3n}\right]y_0^{1+2k+l}+\frac{3bc^2}{3!\,n^3d^3}\left[\frac{1+k+2l-n}{1+k+2l-3n}\right]y_0^{1+k+2l}+\cdots$$

$$+\frac{b^4}{4!\,n^4d^4}\left[\frac{1+4k-n}{1+4k-4n}\right]y_0^{1+4k}+\frac{4b^3c}{4!\,n^4d^4}\left[\frac{1+3k+l-n}{1+3k+l-4n}\right]y_0^{1+3k+l}+\frac{4b^2c^2}{4!\,n^4d^4}\left[\frac{1+2k+2l-n}{1+2k+2l-4n}\right]y_0^{1+2k+2l}+\cdots$$

$$+\frac{b^5}{5!\,n^5d^5}\left[\frac{1+5k-n}{1+5k-5n}\right]y_0^{1+5k}+\frac{5b^4c}{5!\,n^5d^5}\left[\frac{1+4k+l-n}{1+4k+l-5n}\right]y_0^{1+4k+l}+\frac{5b^3c^2}{5!\,n^5d^5}\left[\frac{1+3k+2l-n}{1+3k+2l-5n}\right]y_0^{1+3k+2l}+\cdots$$

$$\vdots \qquad \vdots \qquad \vdots$$

16. The infinite series composed of the terms of the left-hand column of the value of y is convergent when

$$\frac{b^n}{a^k d^{n-k}} \leqq \frac{n^n}{k^k (n-k)^{n-k}}, \tag{28}$$

and if condition (28) is satisfied this infinite series furnishes the solution of the three-term equation

$$ay^n + by^k + d = 0. \tag{29}$$

It is found that each one of the infinite series composed of the terms of the respective columns of (27) is convergent when (28) is satisfied. It follows that (27) may be written

$$y = X_0 + \frac{c}{nd} y_0^{\,l} X_1 + \frac{c^2}{n^2 d^2} y_0^{\,2l} X_2 + \frac{c^3}{n^3 d^3} y_0^{\,3l} X_3 + \cdots, \tag{29}$$

where X_0, X_1, X_2, X_3, $\cdots$, stand for the sums of convergent series. If now X is the largest of the numbers X_0, X_1, X_2, X_3, $\cdots$,

$$y \leqq X\left(1 + \frac{c}{nd} y_0^{\,l} + \frac{c^2}{n^2 d^2} y_0^{\,2l} + \frac{c^3}{n^3 d^3} y_0^{\,3l} + \cdots\right), \tag{30}$$

and this last value of y is convergent when

$$\frac{c}{nd} y_0^{\,l} < 1. \tag{31}$$

Affecting both sides of this inequality by the exponent n, this convergency condition may be written

$$\frac{c^n}{a^l d^{n-l}} < n^n. \tag{32}$$

17. Conditions (28) and (32) are sufficient for the absolute convergence of (27). Condition (28) shows that the series which determines the roots of the three-term equation

$$ay^n + by^k + d = 0 \tag{29}$$

is found from

$$ay^n + by^k x + d = 0. \tag{33}$$

The columns of (27) after the first are the corrections which must be applied to the roots of the three-term equation (29) to obtain the roots of the four-term equation

$$ay^n + by^k + cy^l + d = 0$$

18. If the two terms in the second row of (27) are interchanged and the consequent changes are made throughout (27), the left-hand column in the resulting value of y is convergent if

$$\frac{c^n}{a^l d^{n-l}} \leqq \frac{n^n}{l^l (n-l)^{n-l}}, \tag{34}$$

and the entire expression for y is convergent if in addition

$$\frac{b^n}{a^k d^{n-k}} < n^n. \tag{35}$$

Conditions (34) and (35) are sufficient for the absolute convergence of the new series for y.

Condition (34) shows that the series which determines the solutions of the three-term equation

$$ay^n + cy^l + d = 0 \tag{36}$$

is found from

$$ay^n + cy^l x + d = 0. \tag{37}$$

This series is the left-hand column of the value of y.

Condition (35) shows that the series of corrections which must be applied to the roots of the three-term equation (36) to obtain the solution of the four-term equation

$$ay^n + by^k + cy^l + d = 0$$

is convergent.

19. From equation (21) by omitting in succession each of the terms containing x are obtained the equations

$$ay^n + by^k x + d = 0 \tag{33}$$

$$ay^n + cy^l x + d = 0 \tag{37}$$

The convergency conditions (28) and (34) may be written from equations (33) and (37) respectively by following the directions (a), (b), (c) of article 13. The left-hand members of the conditions (32) and (35), together with the character of the signs of inequality, may be written from equations (37) and (33) respectively by following the same directions. The right-hand member of conditions (32) and (35) is formed by writing the difference of the exponents of the two terms of (21) which do not contain x and

giving this difference an exponent equal to itself. It will be found that when the sign of inequality is $>$ in convergency conditions corresponding to conditions (32) and (35) the right-hand member is the reciprocal of what it is when the sign of inequality is $<$.

20. In like manner two sets of conditions sufficient for the absolute convergence of the infinite series giving the roots of the four-term equation obtained from each one of the equations (21), (22), (23), (24), (25), (26) may be written.

The convergency conditions for all these infinite series may be taken from the following table, in which the signs of equality of the limiting conditions of convergence have been omitted.

(38)	$\frac{b^n}{a^k d^{n-k}}$	$\frac{c^n}{a^l d^{n-l}}$	$\frac{b^{n-l}}{a^{k-l} c^{n-k}}$	$\frac{c^k}{b^l d^{k-l}}$	
(21) $ay^n+by^k x+cy^l x+d=0$	$<$	$<$			n^n
(22) $ay^n+by^k+cy^l x+dx=0$	$>$		$>$		$(n-k)^{n-k}$
(23) $ay^n x+by^k+cy^l x+d=0$	$>$			$<$	k^k
(24) $ay^n x+by^k+cy^l+dx=0$			$>$	$>$	$(k-l)^{k-l}$
(25) $ay^n x+by^k x+cy^l+d=0$		$>$		$>$	l^l
(26) $ay^n+by^k x+cy^l+dx=0$			$<$	$>$	$(n-l)^{n-l}$
	$\frac{n^n}{k^k(n-k)^{n-k}}$	$\frac{n^n}{l^l(n-l)^{n-l}}$	$\frac{(n-l)^{n-l}}{(k-l)^{k-l}(n-k)^{n-k}}$	$\frac{k^k}{l^l(k-l)^{k-l}}$	

In this table the signs of the two inequalities which constitute the convergency conditions of the series obtained from the equations (21) to (26) are placed to the right of the respective equations. The left-hand member of each inequality is at the top of the column in which the sign of inequality stands. The right-hand member of one inequality must be taken at the bottom of the column in which the sign of inequality stands; the right-hand member of the second inequality is the expression at the right of the row in which the sign of inequality stands when the sign of inequality is $<$, when the sign of inequality is $>$ the right-hand member of the inequality is the reciprocal of this expression.

21. The following table exhibits one set of convergency conditions of the infinite series which give the roots of the three-term equation

$$ay^n + by^k + d = 0$$

together with the equations from which these series are derived and

the number of the roots given by each series, and also the conditions sufficient for the absolute convergence of the series of corrections which must be applied to the roots of this three-term equation to obtain the roots of the four-term equation

$$ay^n + by^k + cy^l + d = 0.$$

(39)

			$\frac{b^n}{a^k d^{n-k}}$	$\frac{c^n}{a^l d^{n-l}}$	$\frac{b^{n-l}}{a^{k-l} c^{n-k}}$	$\frac{c^k}{b^l d^{k-l}}$
I	$ay^n+by^kx+d=0$	n	$<$	$<$		
II	$ay^n+by^k+dx=0$	$n-k$	$>$		$>$	
	$ay^nx+by^k+d=0$	k	$>$			$<$
			$\frac{n^n}{k^k(n-k)^{n-k}}$	n^n	$\frac{1}{(n-k)^{n-k}}$	k^k

22. The substitution

(40) $$y = z^s,$$

where s is a positive integer, transforms the four-term equation

$$ay^n + by^k + cy^l + d = 0$$

into the four-term equation

(41) $$az^{ns} + bz^{ks} + cz^{ls} + d = 0.$$

The table of convergency conditions for equation (41) corresponding to table (39) is

(42)

			$\left(\frac{b^n}{a^k d^{n-k}}\right)^s$	$\left(\frac{c^n}{a^l d^{n-l}}\right)^s$	$\left(\frac{b^{n-l}}{a^{k-l} c^{n-k}}\right)^s$	$\left(\frac{c^k}{b^l d^{k-l}}\right)^s$
I	$az^{ns}+bz^{ks}x+d=0$	ns	$<$	$<$		
II	$az^{ns}+bz^{ks}+dx=0$	$ns-ks$	$>$		$>$	
	$az^{ns}x+bz^{ks}+d=0$	ks	$>$			$<$
			$\left(\frac{n^n}{k^k(n-k)^{n-k}}\right)^s$	$(sn)^{sn}$	$\frac{1}{(sn-sk)^{sn-sk}}$	$(sk)^{sk}$

The three-term equations

$$ay^n + by^k + d = 0$$

$$az^{ns} + bz^{ks} + d = 0$$

for all values of s have the same convergency conditions.

If the inequality

$$\frac{c^n}{a^l d^{n-l}} < n^n$$

of table (39) is not satisfied, it is always possible to take s sufficiently large so that the corresponding inequality

$$\frac{c^n}{a^l d^{n-l}} < s^n n^n$$

of table (42) will be satisfied.

In like manner, if the inequalities

$$\frac{b^n}{a^k d^{n-k}} > \frac{n^n}{k^k (n-k)^{n-k}}, \quad \frac{b^{n-l}}{a^{k-l} c^{n-k}} > \frac{1}{(n-k)^{n-k}}, \quad \frac{c^k}{b^l d^{k-l}} < k^k$$

of table (39) are not satisfied simultaneously, it is always possible to take s sufficiently large so that the corresponding inequalities of table (42)

$$\frac{b^n}{a^k d^{n-k}} > \frac{n^n}{k^k (n-k)^{n-k}}, \quad \frac{b^{n-l}}{a^{k-l} c^{n-k}} > \frac{1}{s^{n-k}(n-k)^{n-k}}, \quad \frac{c^k}{b^l d^{k-l}} < s^k k^k$$

will be satisfied simultaneously.

To the convergency conditions of table (42) must be added the limiting convergency conditions obtained by replacing in the first column of inequality signs of table (42) each inequality sign by the equality sign.

It follows that it is always possible to determine s so that all the roots of the four-term equation

$$az^{ns} + bz^{ks} + cz^{ls} + d = 0 \tag{41}$$

may be derived from the roots of the three-term equation

$$az^{ns} + bz^{ks} + d = 0. \tag{42}$$

The roots of the four-term equation

$$ay^n + by^k + cy^l + d = 0$$

are found from the roots of equation (41) by substituting in

$$y = z^s. \tag{40}$$

23. While table (42) shows the possibility of expressing all the

roots of equation (41) in infinite series, the method of article (22) requires the determination of the ns roots of equation (41) to find the n roots of the four-term equation

$$ay^n + by^k + cy^l + d = 0.$$

This method is therefore to be avoided in practice when possible.

The following table exhibits the conditions sufficient for the absolute convergence of the infinite series which give the roots of the four-term equation obtained from the four groups of equations. The series obtained from each group of equations determine all the roots of the four-term equation. The convergency conditions must be taken from this table as in article 20, and the limiting convergency conditions must be taken into account.

A less inclusive set of conditions sufficient for the absolute convergence of the series which give the roots of the four-term equation derived from the groups of equations of table (43) is obtained by taking the second member of each inequality from the bottom of the column in which the sign of inequality stands.

(43)

			$\frac{b^n}{a^k d^{n-k}}$	$\frac{c^n}{a^l d^{n-l}}$	$\frac{b^{n-l}}{a^{k-l} c^{n-k}}$	$\frac{c^k}{b^l d^{k-l}}$	
I	$ay^n+by^k+cy^l+d=0$	n	$<$	$<$			n^n
II	$ay^n+by^k+cy^l+dx=0$	$n-k$	$>$		$>$		$(n-k)^{n-k}$
	$ay^n+by^k+cy^l+d=0$	k	$>$			$<$	k^k
III	$ay^n+by^k+cy^l+dx=0$	$n-l$		$>$	$<$		$(n-l)^{n-l}$
	$ay^n+by^k+cy^l+d=0$	l		$>$		$>$	l^l
IV	$ay^n+by^k+cy^l+dx=0$	$n-k$	$>$		$>$		$(n-k)^{n-k}$
	$ay^n+by^k+cy^l+dx=0$	$k-l$			$>$	$>$	$(k-l)^{k-l}$
	$ay^n+by^k+cy^l+d=0$	l		$>$		$>$	l^l
			$\frac{n^n}{k^k(n-k)^{n-k}}$	$\frac{n^n}{l^l(n-l)^{n-l}}$	$\frac{(n-l)^{n-l}}{(k-l)^{k-l}(n-k)^{n-k}}$	$\frac{k^k}{l^l(k-l)^{k-l}}$	

It is only when the convergency conditions of the groups I, II, III, IV, together with the corresponding limiting convergency conditions fail simultaneously that the use of equation (41) becomes necessary.

IV. The Five-term Equation.

24. In the five-term equation

$$ay^n + by^k + cy^l + dy^m + l = 0$$

the two terms from which the factor x is to be omitted can be selected in ten different ways. This gives rise to the ten equations:

$$(44) \qquad ay^n + by^k x + cy^l x + dy^m x + l = 0$$

$$(45) \qquad ay^n + by^k + cy^l x + dy^m x + lx = 0$$

$$(46) \qquad ay^n + by^k x + cy^l + dy^m x + lx = 0$$

$$(47) \qquad ay^n + by^k x + cy^l x + dy^m + lx = 0$$

$$(48) \qquad ay^n x + by^k + cy^l + dy^m x + lx = 0$$

$$(49) \qquad ay^n x + by^k + cy^l x + dy^m + lx = 0$$

$$(50) \qquad ay^n x + by^k x + cy^l + dy^m + lx = 0$$

$$(51) \qquad ay^n x + by^k + cy^l x + dy^m x + l = 0$$

$$(52) \qquad ay^n x + by^k x + cy^l + dy^m x + l = 0$$

$$(53) \qquad ay^n x + by^k x + cy^l x + dy^m + l = 0$$

Each one of these ten equations defines y as an algebraic function of x which may be expanded into a power series by any one of the methods of articles 7, 8, 9.

25. The terms of the power series expressing the value of the algebraic function defined by equation (44), using the symbol (14) and placing $y_0 = (-l/a)^{1/n}$, when x is made unity, may be arranged as follows:

$$
\begin{aligned}
(54)\quad y = {} & y_0 \\
& + \frac{b}{en}\, y_0^{1+k} \qquad + \frac{c}{en}\, y_0^{1+l} \\
& + \frac{b^2}{2!\, e^2 n^2}\left[1+2k-n\right] y_0^{1+2k} \qquad + \frac{2bc}{2!\, e^2 n^2}\left[1+k+l-n\right] y_0^{1+k+l} \qquad + \frac{c^2}{2!\, e^2 n^2}\left[1+2l-n\right] y_0^{1+2l} \\
& + \frac{b^3}{3!\, e^3 n^3}\left[\frac{1+3k-n}{1+3k-3n}\right] y_0^{1+3k} \qquad + \frac{3b^2 c}{3!\, e^3 n^3}\left[\frac{1+2k+l-n}{1+2k+l-3n}\right] y_0^{1+2k+l} \qquad + \frac{3bc^2}{3!\, e^3 n^3}\left[\frac{1+k+2l-n}{1+k+2l-2n}\right] y_0^{1+k+2l} + \cdots \\
& \qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \\
& + \frac{d}{en}\, y_0^{1+m} \\
& + \frac{2bd}{2!\, e^2 n^2}\left[1+k+m-n\right] y_0^{1+k+m} \qquad + \frac{2cd}{2!\, e^2 n^2}\left[1+l+m-n\right] y_0^{1+l+m} \\
& + \frac{3b^2 d}{3!\, e^3 n^3}\left[\frac{1+2k+m-n}{1+2k+m-3n}\right] y_0^{1+2k+m} + \frac{3bcd}{3!\, e^3 n^3}\left[\frac{1+k+l+m-n}{1+k+l+m-3n}\right] y_0^{1+k+l+m} + \frac{3c^2 d}{3!\, e^3 n^3}\left[\frac{1+2l+m-n}{1+2l+m-3n}\right] y_0^{1+2l+m} + \cdots \\
& \qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots \qquad\qquad\qquad\qquad \vdots
\end{aligned}
$$

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26. The first group of terms of (54) is the infinite series which gives the solution of the four-term equation

$$ay^n + by^k + cy^l + l = 0$$

obtained from the equation

$$ay^n + by^k x + cy^l x + l = 0$$

provided the conditions

$$\frac{b^n}{a^k e^{n-k}} < \frac{n^n}{k^k (n-k)^{n-k}}, \quad \frac{c^n}{a^l e^{n-l}} < n^n,$$

are satisfied.

The second group of terms has the common factor

$$\frac{d}{en} y_0^m,$$

and the successive groups of terms respectively the common factors

$$\frac{d^2}{e^2 n^2} y_0^{2m}, \quad \frac{d^3}{e^3 n^3} y_0^{3m}, \quad \frac{d^4}{e^4 n^4} y_0^{4m}, \quad \cdots.$$

The convergency conditions of the successive groups of terms are identical with the convergency conditions of the first group. It follows that (54) may be written

$$(55) \quad y = Y_0 + Y_1 \frac{d}{en} y_0^m + Y_2 \frac{d^2}{e^2 n^2} y_0^{2m} + Y_3 \frac{d^3}{e^3 n^3} y_0^{3m} + \cdots,$$

where $Y_0, Y_1, Y_2, Y_3, Y_4, \cdots$, represent the sums of convergent infinite series.

If Y denotes the largest of the numbers $Y_0, Y_1, Y_2, Y_3, \cdots$,

$$(56) \quad y \leqq Y \left(1 + \frac{d}{en} y_0^m + \frac{d^2}{e^2 n^2} y_0^{2m} + \frac{d^3}{e^3 n^3} y_0^{3m} + \cdots \right).$$

The series (56) is convergent provided

$$(57) \quad \frac{d}{en} y_0^m < 1.$$

If both members of the inequality (57) are affected by the exponent n, condition (57) becomes

$$(58) \quad \frac{d^n}{a^m e^{n-m}} < n^n.$$

The conditions sufficient for the absolute convergence of (54) are therefore

$$(59)\qquad \frac{b^n}{a^k e^{n-k}} < \frac{n^n}{k^k (n-k)^{n-k}}, \quad \frac{c^n}{a^l e^{n-l}} < n^n, \quad \frac{d^n}{a^m e^{n-m}} < n^n.$$

27. When the conditions (59) are satisfied the first group of terms of (54) gives the roots of the four-term equation

$$ay^n + by^k + cy^l + l = 0$$

expressed in the series obtained from the equation

$$ay^n + by^k x + cy^l x + l = 0$$

and the successive groups of (54) are the series of corrections which must be applied to the roots of this four-term equation to obtain the roots of the five-term equation

$$ay^n + by^k + cy^l + dy^m + l = 0.$$

28. If in the first row of (54) either of the terms

$$\frac{c}{en} y_0^{1+l}, \quad \frac{d}{en} y_0^{1+m}$$

is placed first and the consequent changes in (54) are made, the convergency conditions of the two new series are found to be

$$(60)\qquad \frac{b^n}{a^k e^{n-k}} < n^n, \quad \frac{c^n}{a^l e^{n-l}} < \frac{n^n}{l^l (n-l)^{n-l}}, \quad \frac{d^n}{a^m e^{n-m}} < n^n;$$

$$(61)\qquad \frac{b^n}{a^k e^{n-k}} < n^n, \quad \frac{c^n}{a^l e^{n-l}} < n^n, \quad \frac{d^n}{a^m e^{n-m}} < \frac{n^n}{m^m (n-m)^{n-m}}.$$

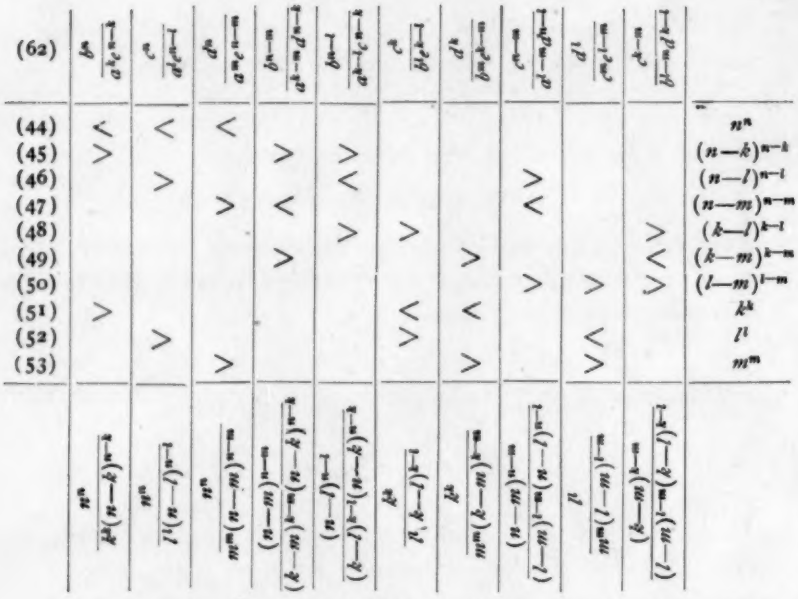
In the limiting convergency conditions the signs of inequality in the first inequality of (59), in the second inequality of (60) and in the third inequality of (61) must be replaced by the equality sign.

The conditions sufficient for the absolute convergence of (54) may be written from equation (44) by the method stated in article 19.

In like manner the conditions sufficient for the absolute convergence of the series obtained from equations (45) to (53) may be written.

The convergency conditions for all these series may be taken from the following table. The convergency conditions are taken from the table by the method stated in article 20, except that the right-hand members of two inequalities must be determined from

the expressions at the right of the row in which the sign of inequality stands.



(62)	$\frac{q-u}{q^q}$	$\frac{l-u}{c^l}$	$\frac{m-u}{a^m}$	$\frac{l-q}{b^{l-q}}$	$\frac{m-q}{a^{m-q}}$	$\frac{l-q}{c^{l-q}}$	$\frac{m-q}{b^{m-q}}$	$\frac{m-l}{a^{m-l}}$	$\frac{l-q}{d^{l-q}}$	$\frac{m-l}{c^{m-l}}$	$\frac{l-q}{b^{l-q}}$	$\frac{m-q}{a^{m-q}}$	
(44)	<	<	<										n^n
(45)	>			>		>				>			$(n-k)^{n-k}$
(46)		>		<		<				<			$(n-l)^{n-l}$
(47)			>	<									$(n-m)^{n-m}$
(48)						>	>					>	$(k-l)^{k-l}$
(49)				>			>					<	$(k-m)^{k-m}$
(50)									>	>	>	<	$(l-m)^{l-m}$
(51)	>						<		>				k^k
(52)		>					>		<				l^l
(53)			>				>		<				m^m
	$\frac{q-u}{u^q}$	$\frac{l-u}{u^l}$	$\frac{m-u}{u^m}$	$\frac{q-u}{u^{q-u}}$	$\frac{m-u}{u^{m-u}}$	$\frac{l-u}{u^{l-u}}$	$\frac{m-u}{u^{m-u}}$	$\frac{m-u}{u^{m-u}}$	$\frac{l-u}{u^{l-u}}$	$\frac{m-u}{u^{m-u}}$	$\frac{l-u}{u^{l-u}}$	$\frac{m-u}{u^{m-u}}$	

29. The following table exhibits one set of conditions sufficient for the absolute convergence of the infinite series which give the roots of the four-term equation

$$ay^n + by^k + cy^l + l = 0$$

together with the equations from which these series are obtained and the number of roots given by each series, and also the conditions sufficient for the absolute convergence of the series of corrections which must be applied to the roots of this four-term equation to obtain the roots of the five-term equation

$$ay^n + by^k + cy^l + dy^m + l = 0.$$

(63)

			$\frac{b^m}{a^k e^{n-k}}$	$\frac{c^n}{a^l e^{n-l}}$	$\frac{b^{n-l}}{a^{k-l} e^{n-k}}$	$\frac{e^k}{b^l e^{k-l}}$	$\frac{d^n}{a^m e^{n-m}}$	$\frac{b^{n-m}}{a^{k-m} d^{n-k}}$	$\frac{d^k}{b^m e^{k-m}}$
I	$ay^n+by^k+cy^l+e=0$	n	<	<			<		
II	$ay^n+by^k+cy^l+ex=0$	$n-k$	>		>			>	
	$ay^n x+by^k+cy^l+e=0$	k	>			<			<
			$\frac{n^n}{k^k(n-k)^{n-k}}$	n^n	$\frac{\text{I}}{(n-k)^{n-k}}$	k^k	n^n	$\frac{\text{I}}{(n-k)^{n-k}}$	k^k

30. The substitution

$$y = z^s, \tag{64}$$

where s is a positive integer, transforms the five-term equation

$$ay^n + by^k + cy^l + dy^m + l = 0$$

into another five-term equation

$$ay^{ns} + by^{ks} + cy^{ls} + dy^{ms} + l = 0. \tag{65}$$

An examination of the table of convergency conditions for equation (65) corresponding to table (63), shows that it is always possible so to determine s that the convergency conditions for the series obtained from the equation

$$ay^{ns} + by^{ks}x + cy^{ls}x + dy^{ms}x + l = 0 \tag{66}$$

or from the pair of equations

$$ay^{ns} + by^{ks} + cy^{ls}x + dy^{ms}x + lx = 0 \tag{67}$$

$$ay^{ns}x + by^{ks} + cy^{ls}x + dy^{ms}x + l = 0 \tag{68}$$

are satisfied. Hence it is always possible to determine all the roots of a five-term equation by means of series.

31. The method of article 30 requires the determination of the ns roots of equation (65) in order to find the n roots of the five-term equation

$$ay^n + by^k + cy^l + dy^m + l = 0.$$

The use of this method becomes necessary only when the convergency conditions of the seven groups of equations of the following table, together with the corresponding limiting convergency conditions fail simultaneously.

The convergency conditions must be taken from this table as in article 20.

A less inclusive set of congruency conditions may be taken from this table as in article 23.

(69)

I
II
III
IV
V
VI
VII

V. Conclusion.

32. In the algebraic equation of t terms

$$f(y)=0$$

the two terms from which the factor x is to be omitted can be selected in

$$\frac{t(t-1)}{2}$$

ways. Each one of the resulting equations defines y as an algebraic function of x, and each algebraic function of x can be expanded into a power series in x by the methods used to obtain the corresponding expansions for the three-, four- and five-term equations. When x is made unity in these power series the resulting series become the roots of the t-term equation and a table of convergency conditions for these series analogous to tables (20), (38), (62) can be set up. In fact, this table may be written mechanically by following the directions of article 19.

33. If in the t-term equation the substitution

$$y=z^s$$

is made, a table of convergency conditions analogous to tables (39), (63) can be set up, and the value of s can be determined so that this table of conditions shows that it is possible to obtain all the roots of the transformed equation from the series derived either from the equation in which x is omitted from the first and last terms, or from the two equations in which x is omitted from the first and second, and from the second and last terms respectively. The roots of the given equation are then found from the roots of the transformed equation by substituting in

$$y=z^s.$$

34. Finally, tables of convergency conditions analogous to tables (43), (64) can be set up for the t-term equation, and it is necessary to use the transformed equation only when the convergency condi-

tions of all the groups of this table, together with the corresponding limiting convergency conditions, fail simultaneously.

35. It follows that all the roots of an algebraic equation of any number of terms, that is, of any algebraic equation, can be expressed in infinite series by the method of this investigation.

LEHIGH UNIVERSITY, BETHLEHEM, PA.,
April 2, 1908.

Stated Meeting May 1, 1908.

Treasurer JAYNE in the Chair.

Dr. Martin G. Brumbaugh, a newly elected member, was presented to the chair, and took his seat in the Society.

Letters were read, accepting election to membership from

Martin Grove Brumbaugh, Ph.D., Philadelphia.
Walter Bradford Cannon, A.M., M.D., Boston, Mass.
James Christie, Philadelphia.
Edward Washburn Hopkins, Ph.D., LL.D., New Haven, Conn.
Josiah Royce, Ph.D., LL.D., Cambridge, Mass.
Jacob G. Schurman, Ph.D., Ithaca, N. Y.
Edward Anthony Spitzka, M.D., Philadelphia.
Robert Williams Wood, Ph.D., Baltimore.

MR. R. H. MATHEWS presented some "Notes on Australian Laws of Descent."

Professor Albert A. Michelson, of Chicago, was unanimously elected a Vice-President to fill the unexpired term of Professor George F. Barker, resigned.

Stated Meeting May 15, 1908.

Curator DOOLITTLE in the Chair.

Letters were read accepting membership from

William Hallock, Ph.D., New York City.
Leonard Pearson, M.D., Philadelphia.
Charles Henry Smyth, Ph.D., Princeton, N. J.
John Robert Sitlington Sterrett, Ph.D., Ithaca, N. Y.
Ernest Nys, Brussels.

From Professor Albert A. Michelson accepting election to the Vice-Presidency to fill an unexpired term.

From the Committee of Organization of the Third Congrès International de Botanique, announcing that the Congress will be held at Brussels from May 14–22, 1910, and inviting the Society to be represented by delegates.

Dr. H. M. Chance read a paper on "The Origin of Bombshell Ore" (see page 135), which was discussed by Mr. Sanders, Mr. Jayne and Professor Doolittle.

THE ORIGIN OF BOMBSHELL ORE.

By H. M. CHANCE.

(*Read May 15, 1908.*)

The term "bombshell" ore is applied by miners and iron-masters to hollow masses of limonite—brown hematite—which sometimes are round or oval but more commonly are of any irregular shape. The "bombs" may contain water, clay, sand, quartz, flint, pyrite, siderite, sandstone or decomposed slate, or may be entirely empty. Geologists usually speak of such ore as nodular or concretionary. A careful examination of the literature of ore deposits and especially of that relating to the genesis of limonite ores fails to disclose a satisfactory explanation of its origin of mode of formation. By many it is assumed to be similar in origin to silicious geodes, which are supposed to be formed by the deposition of silica or silicates upon the walls of cavities, while others describe it as of "concretionary" origin without attempting to explain the process of formation or the manner in which it has occluded the variety of materials which are found in the interior of different specimens from the same locality. That it is not of concretionary origin is evident upon even cursory consideration, for concretions are masses of material arranged in concentric layers around a central nucleus. The latter may be a grain of sand, a pebble, fossil, or any substance around which (as a core) the concretion forms, growing from the center by the successive addition of concentric rings. Concretions are perhaps merely symmetrical segregations.

The peculiarities of this ore are well described by Professor T. C. Hopkins, *Bull. Geol. Soc. Am.*, 1890, Vol. 11, p. 477, etc., as follows:

> "Nodular ore consists of irregularly rounded masses, varying in size from a fraction of a pound to several hundred pounds in weight. The masses are frequently hollow, but some enclose a rounded or sub-angular rock fragment, which is sometimes sandstone, . . . sometimes chert, sometimes slate, and sometimes clay. Some shells are filled with clay and sand,

and workmen report finding many of them filled with water. Some are filled with clay, which still retains the laminated structure and appearance of the original slate from which the clay was derived, furthermore, the slaty structure was found to extend through the ore shell, which showed, besides the plain lamination of slate, a faint concentric structure as well. . . . While only one shell was found still retaining the laminations of the clay, there were many others containing clay and sand. Some of the shells were but thin crusts, while others were quite thick, almost solid; some have a smooth, velvety or bright mammillated inner surface, frequently coated with manganese oxide. In some instances the lining of the shell is covered with many small stalactites of ore. . . . Many of the shells are lined with a dense fibrous layer, often an inch or more in thickness. . . . The thinner shells have all been broken, and we see only the fragments of them in the clay-ore masses. This shell form of ore . . . forms an appreciable part of the ore body in many cases. The small, irregular, nodular-like pieces of ore, commonly knows as shot ore, are presumably closely related in origin to the shells. . . ."

The inner wall of many bombs consists of a hard, bright, brown or jet-black, glazed surface, curved, rounded or botryoidal. This is frequently described as a manganese coating, but is doubtless a film of iron or manganese silicate. Occasionally the interior or a part of the interior is lined with a layer of extremely hard, flinty, liver-colored iron silicate, or with quartz crystals or chalcedony, and the same silicate frequently forms a considerable portion of the body of the shell or of its outer layers, but generally the shell is composed of high-grade limonite, of a fibrous structure, especially in those layers forming the inner lining of the shell.

These peculiarities are satisfactorily accounted for neither by the theory that these ore masses owe their origin to concretionary action, nor by that which assumes the direct deposition of ferric hydrate upon the interior of rock (limestone?) cavities. They may, however, be explained by assuming that the bombs are the residual masses, remaining after oxidation, of iron sulphides or carbonates containing sand or clay or both in varying proportions.

If the material from which this ore is formed consists of sandstone, or of sandy slate, or of clay slate, impregnated more or less completely with pyrite or siderite, the formation of bombshells, containing just such materials as are found in these shells, may be readily explained, especially if the iron impregnation be in the form of pyrite or marcasite, that is, FeS_2.

If such sandstone or slate, is broken and fissured by faulting and crushing, and by the development of cleavage planes, oxidation by percolating waters will proceed along the joints or planes which form the channels through which these waters circulate, and in each fragment of the mineralized rock oxidation will commence upon the outside and progress towards the center.

In this way on outer skin or shell of limonite first forms on the outside of the fragment, for if the iron be present as pyrite or marcasite while some of it may be removed as ferrous sulphate, this salt, if formed, may immediately be oxidized and precipitated *in situ* as ferric hydrate. The sulphuric acid formed by the oxidation of the remaining molecule of sulphur will attack and decompose the clay of the gangue, removing the bases as sulphates in solution; the silicic acid also escaping in solution, or combining with iron oxides to form iron silicates, remains as an integral part of the ore.

If clay be present in large quantity a portion will remain undecomposed in the center of the bomb, together with all of the sand originally present in the gangue.

Hence, if the original pyritic material has a clayey (slate) gangue, bombs may form containing no residual matter, or containing more or less clay; if the gangue be sand and clay (arenaceous slate), the sand only, or sand and clay may remain; if the gangue be sand only, some of this will remain as an impurity in the limonite forming the body of the shell, and some as a partial filling of the interior of the bomb.

It is now well known that pyrite (or marcasite) oxidizing underground, whether by waters carrying free oxygen, or by waters containing no uncombined oxygen, or by reactions involving hydrolysis, does not behave in the same way as when oxidized by exposure to the air above-ground. One of the most common reactions above-ground is that in which sulphur is set free, often written:

$$FeS_2 + 4O = FeSO_4 + S,$$

but this rarely occurs beneath the surface, for the gossans of pyritic veins seldom carry free sulphur, although there are a few noted examples in which large deposits of sulphur are found between the surface and the unoxidized portions of such veins.

In the absence of oxygen, carbonates in solution may, as shown by Dr. N. H. Stokes, completely oxidize the iron of pyrite or marcasite thus:

$$8FeS_2 + 15Na_2CO_3 = 4Fe_2O_3 + 14Na_2S + Na_2S_2O_3 + 15CO_2,$$

and under proper conditions of temperature and pressure the ferric oxide thus formed may be deposited as hydrate; but these reactions do not satisfy the observed conditions and it seems more probable that oxidation near the surface has proceeded as indicated by some of the following reactions:

$$FeS_2 + H_2O + 7O = FeSO_4 + H_2SO_4;$$
$$3FeS_2 + 8H_2O + 22O = Fe_2O_3,\ 3H_2O + FeSO_4 + 5H_2SO_4;$$
$$2FeSO_4 + 5H_2O + O = Fe_2O_3,\ 3H_2O + 2H_2SO_4;$$
$$2FeS_2 + 7H_2O + 15O = Fe_2O_3,\ 3H_2O + 4H_2SO_4;$$
$$2FeS_2 + 4H_2O + 15O = Fe_2O_3 + 4H_2SO_4;$$
$$2FeSO_4 + 2H_2O + O = Fe_2O_3 + 2H_2SO_4;$$
$$6FeSO_4 + 3O = Fe_2O_3 + 2Fe_2(SO_4)_3;$$
$$6FeSO_4 + 3H_2O + 3O = Fe_2O_3,\ 3H_2O + 2Fe_2(SO_4)_3;$$
$$2Fe_2(SO_4)_3 + 2FeS_2 + 4H_2O + 12O = 6FeSO_4 + 4H_2SO_4.$$

The sulphuric acid having been formed in direct contact with the gangue, it is reasonable to suppose that it must at once attack any clay or other decomposable material, and the removal of the soluble silicates and silicic acid by transfusion through the walls of the bomb is readily pictured. It is, however, possible that the colloidal silicic acid may be retained, and further that it may perhaps often be set free in a gelatinous condition. This latter hypothesis may account for the frequent presence in such ores of a skeleton of amorphous silica which appears to completely ramify some parts of the limonite.

If the oxidation proceed according to these equations, the successive additions of layers of limonite to the interior of the shells is doubtless due to the further oxidation of the ferrous sulphate as above shown, the oxidation of the solution occurring at or in the wall of the shell where the solutions, in escaping by transfusion through the walls of the shell, are met by oxidizing waters transfusing towards the center of the shell. Under such conditions the ferric

hydrate would be deposited in the pores of the shell or upon its inner surface.

In attempting to picture these reactions and their results, it is important to remember the extremely slow rate at which oxidation proceeds under such conditions. Even at the surface where decomposition is comparatively rapid, the oxidation of pyrite appears to progress at a very slow rate, perhaps not exceeding an inch or a few inches in depth in several hundred years.

If the iron be present as carbonate, a precisely similar series of reactions may be conceived, in which carbonic acid transposes the silicates, freeing silicic acid and removing the bases as soluble carbonates.

Other observers have noted the occasional presence of a central core of siderite or pyrite in bombshell ore, but have generally attributed the presence of such cores to concretionary action and replacement by sulphates (accompanied by reduction to sulphide) or carbonates in solution.

The foregoing theory, advanced to account for the origin of bombshell ore, is based upon a study of these deposits dating back to 1885—when the writer was personally engaged in mining brown-hematite ore—and upon examinations of many specimens which show more or less clearly the character of the original material from which such ore is formed. It will form an integral part of a broader statement, extending the application of this theory to the genesis of limonite ores, and including a discussion of the original sources of the iron, methods of mineralization, and subsequent decomposition and precipitation.